

# YEAR 8 - ALGEBRAIC TECHNIQUES...

## Sequences

### What do I need to be able to do?

By the end of this unit you should be able to:

- Generate a sequence from term to term or position to term rules
- Recognise arithmetic sequences and find the  $n$ th term
- Recognise geometric sequences and other sequences that arise

### Keywords

**Sequence:** items or numbers put in a pre-decided order

**Term:** a single number or variable

**Position:** the place something is located

**Linear:** the difference between terms increases or decreases (+ or -) by a constant value each time

**Non-linear:** the difference between terms increases or decreases in different amounts, or by  $\times$  or  $\div$

**Difference:** the gap between two terms

**Arithmetic:** a sequence where the difference between the terms is constant

**Geometric:** a sequence where each term is found by multiplying the previous one by a fixed non zero number

### Linear and Non Linear Sequences

**Linear Sequences** – increase by addition or subtraction and the same amount each time

**Non-linear Sequences** – do not increase by a constant amount – quadratic, geometric and Fibonacci

- Do not plot as straight lines when modelled graphically
- The differences between terms can be found by addition, subtraction, multiplication or division

**Fibonacci Sequence** – look out for this type of sequence

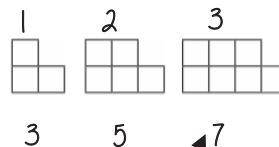
0 1 1 2 3 5 8 ...

Each term is the sum of the previous two terms.



### Sequence in a table and graphically

**Position:** the place in the sequence



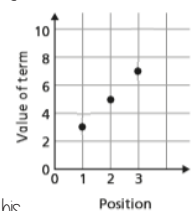
**Term:** the number or variable (the number of squares in each image)

**In a table**

Position	1	2	3
Term	3	5	7

+2 +2

**Graphically**



Because the terms increase by the same addition each time this is **linear** – as seen in the graph

### Sequences from algebraic rules

This is substitution!

$$3n + 7$$

This will be linear - note the single power of  $n$ . The values increase at a constant rate

$$2n - 5$$

Substitute the number of the term you are looking for in place of 'n'

e.g.

$$1^{\text{st}} \text{ term} = 2(1) - 5 = -3$$

$$2^{\text{nd}} \text{ term} = 2(2) - 5 = -1$$

$$100^{\text{th}} \text{ term} = 2(100) - 5 = 195$$

$$3n^2 + 7$$

This is not linear as there is a power for  $n$

### Checking for a term in a sequence

Form an equation

Is 201 in the sequence  $3n - 4$ ?

$$3n - 4 = 201$$

Algebraic rule

Term to check

Solving this will find the position of the term in the sequence. ONLY an integer solution can be in the sequence.

### Complex algebraic rules

Misconceptions and comparisons

$$2n^2$$

2 times whatever  $n$  squared is

e.g.

$$1^{\text{st}} \text{ term} = 2 \times 1^2 = 2$$

$$2^{\text{st}} \text{ term} = 2 \times 2^2 = 8$$

$$100^{\text{th}} \text{ term} = 2 \times 100^2 = 20000$$

$$(2n)^2$$

2 times  $n$  then square the answer

e.g.

$$1^{\text{st}} \text{ term} = (2 \times 1)^2 = 4$$

$$2^{\text{st}} \text{ term} = (2 \times 2)^2 = 16$$

$$100^{\text{th}} \text{ term} = (2 \times 100)^2 = 40000$$

$$n(n + 5)$$

e.g.

$$1^{\text{st}} \text{ term} = 1(1 + 5) = 6$$

$$2^{\text{st}} \text{ term} = 2(2 + 5) = 14$$

$$100^{\text{th}} \text{ term} = 100(100 + 5) = 10500$$

You don't need to expand the expression

### Finding the algebraic rule

This is the 4 times table  $\rightarrow 4, 8, 12, 16, 20, \dots$

$$4n$$

$$7, 11, 15, 19, 22$$

This has the same constant difference – but is 3 more than the original sequence

$$4n + 3$$

$$4n + 3$$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

# YEAR 8 - REASONING WITH ALGEBRA...

## Straight Line Graphs

### What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use  $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

### Keywords

**Gradient:** the steepness of a line

**Intercept:** where two lines cross. The y-intercept: where the line meets the y-axis.

**Parallel:** two lines that never meet with the same gradient

**Co-ordinate:** a set of values that show an exact position on a graph

**Linear:** linear graphs (straight line) – linear common difference by addition/ subtraction

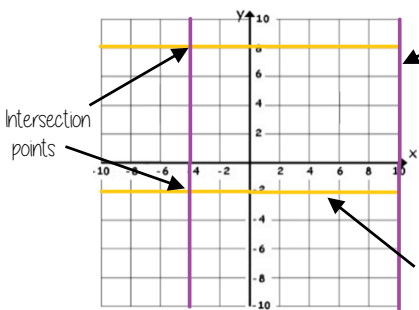
**Asymptote:** a straight line that a graph will never meet

**Reciprocal:** a pair of numbers that multiply together to give 1

**Perpendicular:** two lines that meet at a right angle.

### Lines parallel to the axes

R



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form  $x = a$  and are vertical

Lines parallel to the x axis take the form  $y = a$  and are horizontal

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

### Plotting $y = mx + c$ graphs

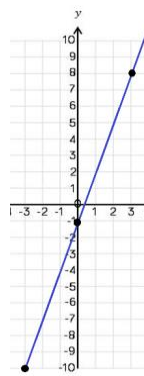
R

$y = 3x - 1$  → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

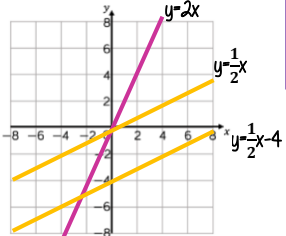
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

### Compare Gradients

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient – the steeper the line

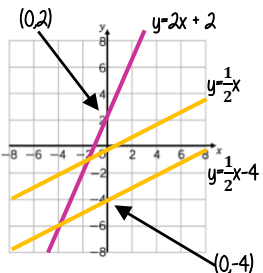
Parallel lines have the same gradient

Positive gradients  
Negative gradients

### Compare Intercepts

$$y = mx + c$$

The value of c is the point at which the line crosses the y-axis. Y intercept



The coordinate of a y intercept will always be (0,c)

Lines with the same y-intercept cross in the same place

$$y = mx + c$$

The coefficient of x (the number in front of x) tells us the gradient of the line

$$y = mx + c$$

y and x are coordinates

The value of c is the point at which the line crosses the y-axis. Y intercept

The equation of a line can be rearranged. Eg

$$y = c + mx$$

$$c = y - mx$$

Identify which coefficient you are identifying or comparing

### Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

The y-intercept shows the minimum charge.  
The gradient represents the price per mile

### Direct Proportion graphs

To represent direct proportion the graph must start at the origin

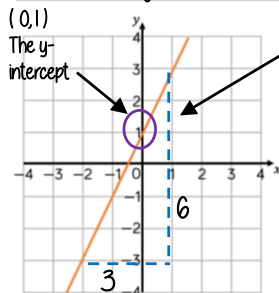
A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost  
The gradient shows the price per pen

### Find the equation from a graph



The Gradient  $\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient

Positive gradients  
Negative gradients

# YEAR 8 - REPRESENTATIONS...

## Working in the Cartesian plane

### What do I need to be able to do?

By the end of this unit you should be able to:

- Label and identify lines parallel to the axes
- Recognise and use basic straight lines
- Identify positive and negative gradients
- Link linear graphs to sequences
- Plot  $y = mx + c$  graphs

### Keywords

**Quadrant:** four quarters of the coordinate plane.

**Coordinate:** a set of values that show an exact position.

**Horizontal:** a straight line from left to right (parallel to the x axis)

**Vertical:** a straight line from top to bottom (parallel to the y axis)

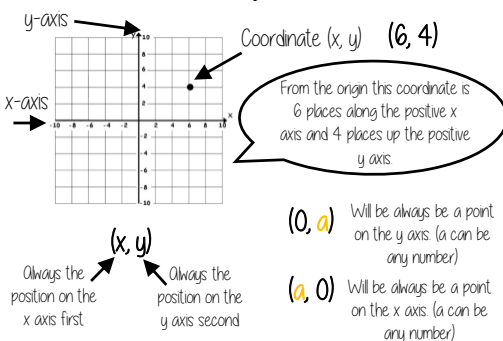
**Origin:** (0,0) on a graph. The point the two axes cross

**Parallel:** Lines that never meet

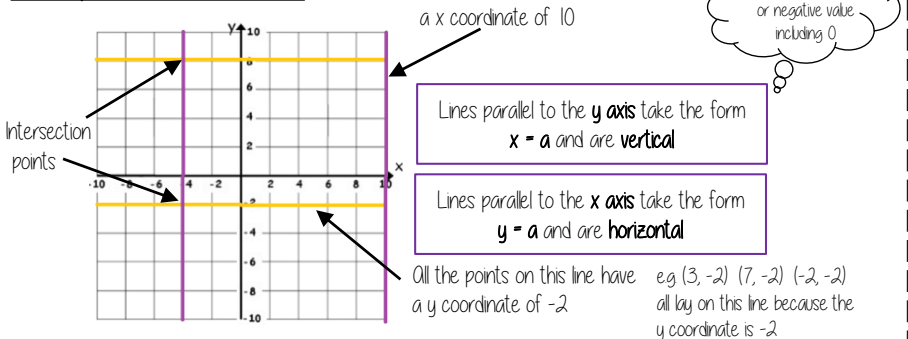
**Gradient:** The steepness of a line

**Intercept:** Where lines cross

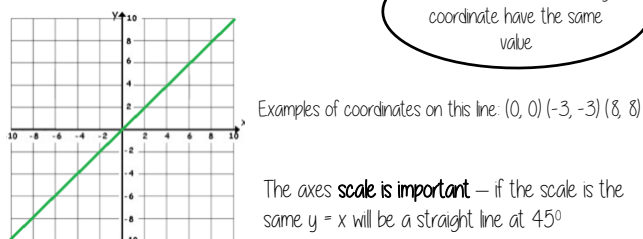
### Coordinates in four quadrants



### Lines parallel to the axes

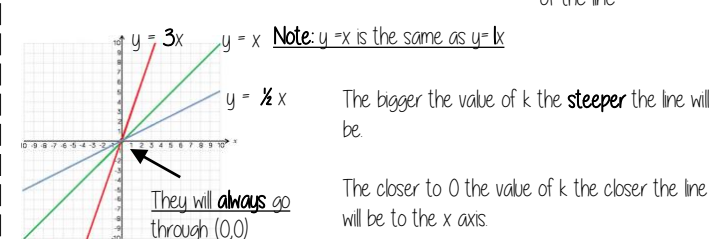


### Recognise and use the line $y = x$

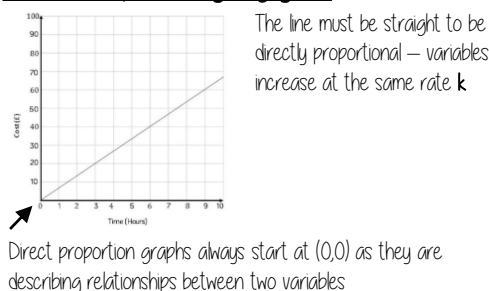


### Recognise and use the lines $y = kx$

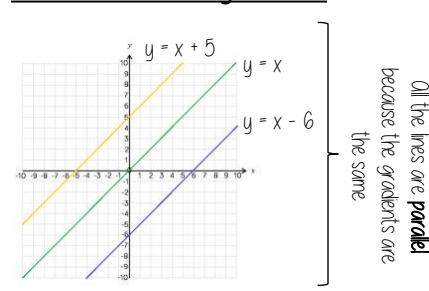
The value of k changes the steepness of the line



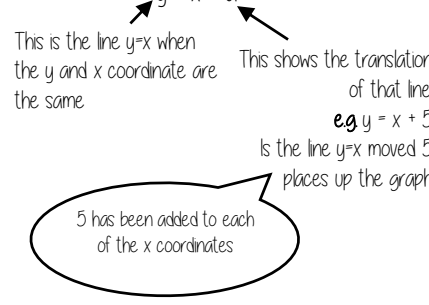
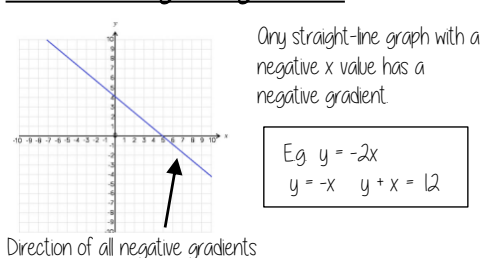
### Direct Proportion using $y = kx$



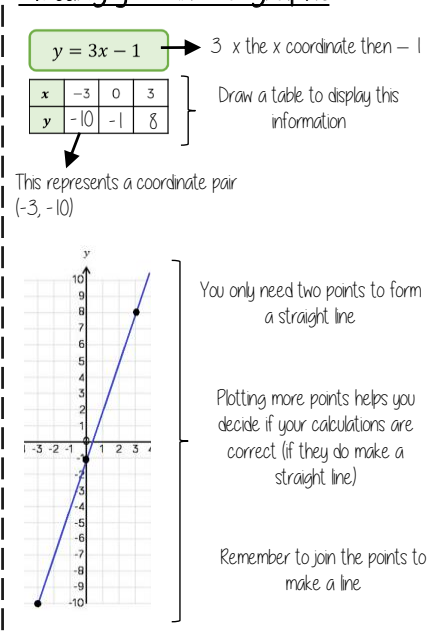
### Lines in the form $y = x + a$



### Lines with negative gradients



### Plotting $y = mx + c$ graphs



# YEAR 8 - DEVELOPING GEOMETRY...

## Angles in parallel lines and polygons

### What do I need to be able to do?

By the end of this unit you should be able to:

- Identify alternate angles
- Identify corresponding angles
- Identify co-interior angles
- Find the sum of interior angles in polygons
- Find the sum of exterior angles in polygons
- Find interior angles in regular polygons

### Keywords

**Parallel:** Straight lines that never meet

**Angle:** The figure formed by two straight lines meeting (measured in degrees)

**Transversal:** A line that cuts across two or more other (normally parallel) lines

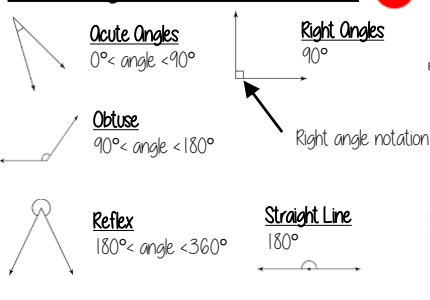
**Isosceles:** Two equal size lines and equal size angles (in a triangle or trapezium)

**Polygon:** A 2D shape made with straight lines

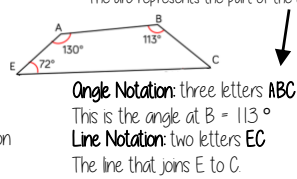
**Sum:** Addition (total of all the interior angles added together)

**Regular polygon:** All the sides have equal length; all the interior angles have equal size.

### Basic angle rules and notation



The letter in the middle is the angle  
The arc represents the part of the angle

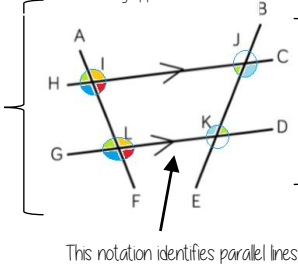


### Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!

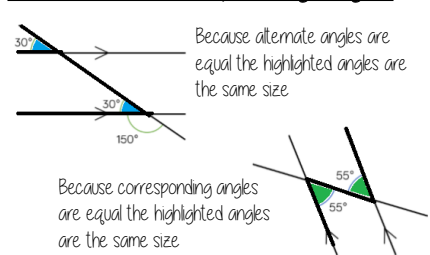
Lines AF and BE are transversals (lines that bisect the parallel lines)

Corresponding angles often identified by their "F shape" in position

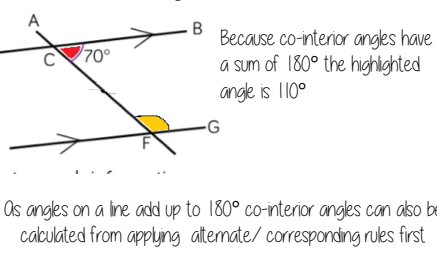


Alternate angles often identified by their "Z shape" in position

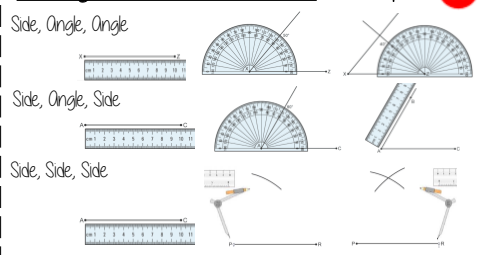
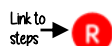
### Alternate/ Corresponding angles



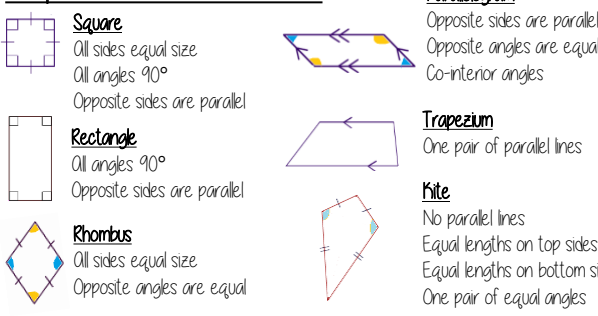
### Co-interior angles



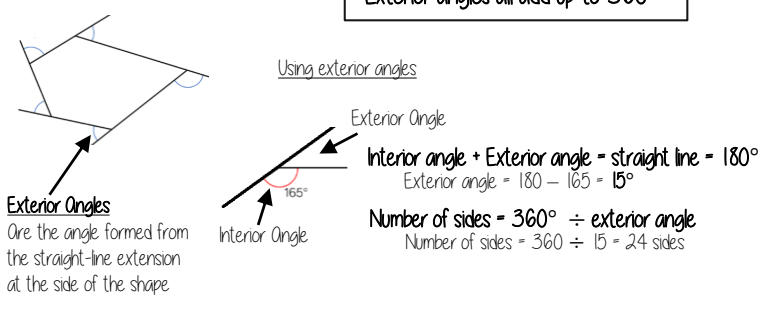
### Triangles & Quadrilaterals



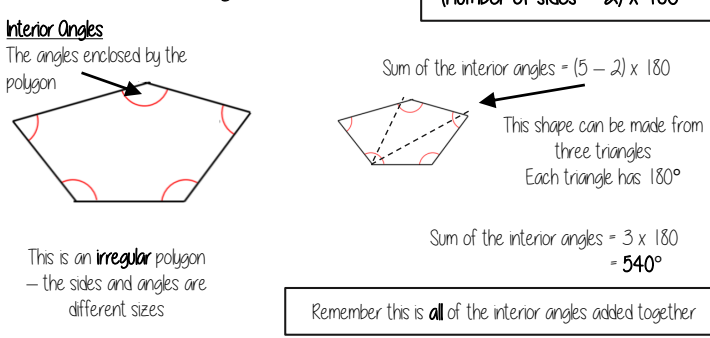
### Properties of Quadrilaterals



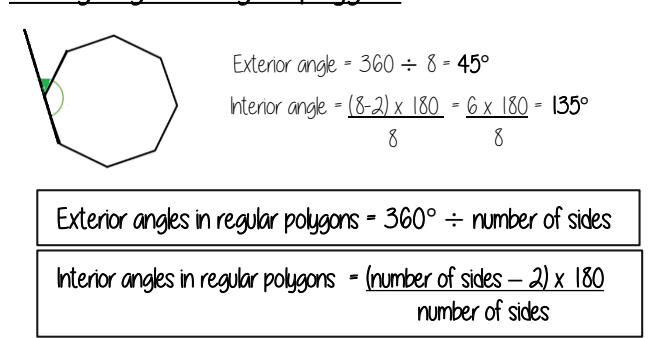
### Sum of exterior angles



### Sum of interior angles



### Missing angles in regular polygons





# YEAR 8 - LINES AND ANGLES

## Constructing, measuring and using geometric notation

### What do I need to be able to do?

By the end of this unit you should be able to:

- Use letter and labelling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

### Keywords

**Polygon:** A 2D shape made with straight lines

**Scalene triangle:** a triangle with all different sides and angles

**Isosceles triangle:** a triangle with two angles the same size and two angles the same size

**Right-angled triangle:** a triangle with a right angle

**Frequency:** the number of times a data value occurs

**Sector:** part of a circle made by two radii touching the centre

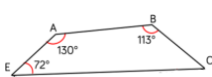
**Rotation:** turn in a given direction

**Protractor:** equipment used to measure angles

**Compass:** equipment used to draw arcs and circles

### Letter and labelling convention

The letter in the middle is the angle  
The arc represents the angle



**Angle Notation:** three letters ABC

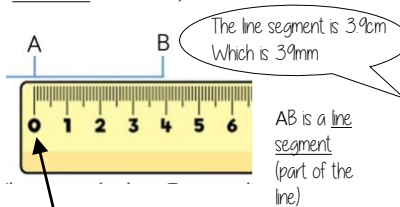
This is the angle at B =  $113^\circ$

**Line Notation:** two letters EC

The line that joins E to C.

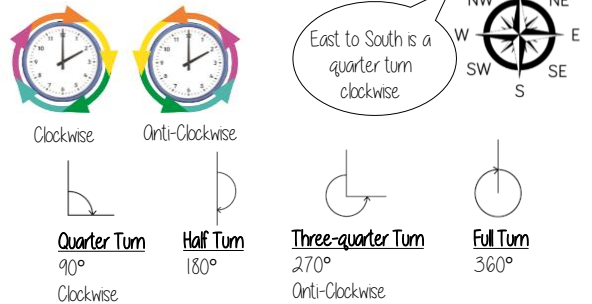
### Draw and measure line segments

Conversions:  $1\text{cm} = 10\text{mm}$ ,  $1\text{m} = 100\text{cm}$

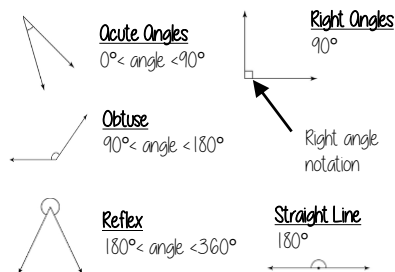


Make sure the start of the line is at 0.

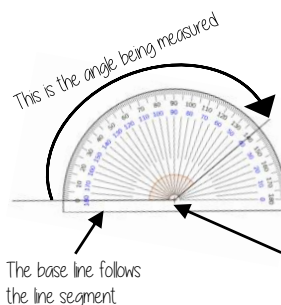
### Angles as measures of turn



### Classify angles

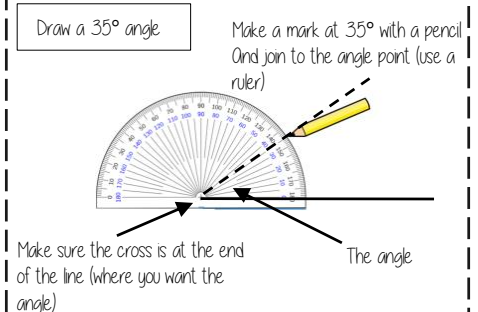


### Measure angles to $180^\circ$



Read from  $0^\circ$  on the base line.  
Remember to use estimation.  
This is an obtuse angle so between  $90^\circ$  and  $180^\circ$

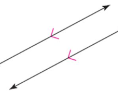
### Draw angles up to $180^\circ$



### Parallel and Perpendicular lines

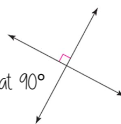
#### Parallel lines

Straight lines that never meet  
(Have the same gradient)



#### Perpendicular lines

Straight lines that meet at  $90^\circ$



### Angles over $180^\circ$

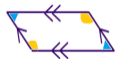
Use your knowledge of straight lines  $180^\circ$  and angles around a point  $360^\circ$

$360^\circ$  - smaller angle = reflex angle



### Properties of Quadrilaterals

**Square**  
All sides equal size  
All angles  $90^\circ$   
Opposite sides are parallel



#### Parallelogram

Opposite sides are parallel  
Opposite angles are equal  
Co-interior angles

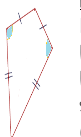
**Rectangle**  
All angles  $90^\circ$   
Opposite sides are parallel



#### Trapezium

One pair of parallel lines

**Rhombus**  
All sides equal size  
Opposite angles are equal



#### Kite

No parallel lines  
Equal lengths on top sides  
Equal lengths on bottom sides  
One pair of equal angles

### Draw Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

$\frac{32}{60}$  "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$$\frac{32}{60} \times 360 = 192^\circ$$

Use a protractor to draw  
This is  $192^\circ$



### Polygons

3	- Triangle	5	- Pentagon	8	- Octagon
4	- Quadrilateral	6	- Hexagon	9	- Nonagon
		7	- Heptagon	10	- Decagon

### SAS, SSS, ASA constructions

Side, Angle, Angle



Side, Angle, Side



Side, Side, Side



If all the sides and angles are the same, it is a **regular** polygon

# YEAR 8 - APPLICATION OF NUMBER

## Solving problems with multiplication and division

### What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and use factors
- Understand and use multiples
- Multiply/ Divide integers and decimals by powers of 10
- Use formal methods to multiply
- Use formal methods to divide
- Understand and use order of operations
- Solve area problems
- Solve problems using the mean

### Keywords

**Array:** an arrangement of items to represent concepts in rows or columns

**Multiples:** found by multiplying any number by positive integers

**Factor:** integers that multiply together to get another number.

**Mil:** prefix meaning one thousandth

**Centi:** prefix meaning one hundredth

**Kilo:** prefix meaning multiply by 1000

**Quotient:** the result of a division

**Dividend:** the number being divided

**Divisor:** the number we divide by

### Factors

Arrays can help represent factors

**Factors of 10**  
1, 2, 5, 10

10 x 1 or 1 x 10

5 x 2 or 2 x 5

The number itself is always a factor

**Square numbers** have an **ODD** number of factors

**Factors of 4**  
1, 2, 4

**Factors of 36**  
1, 2, 3, 4, 6, 9, 12, 18, 36

Be strategic  
- Lay factors out in pairs can help you not to miss any

### Multiples

Bar models can represent by something is a multiple. Eg. 20 is a multiple of 4

#### Lowest Common Multiples

**9** 9, 18, 27, 36, 45, 54

**12** 12, 24, 36, 48, 60

**LCM of 9 and 12**

The first time their multiples match

**LCM = 36**



### Multiply/ Divide by powers of 10

100s 10s 1s

$\times 100$

$3 \times 100 = 300$

1s  $\frac{1}{10}$   $\frac{1}{100}$

$\times 100$

$0.03 \times 100 = 3$

Repeated multiplication and division by powers of 10 is commutative

$\div 10$  then  $\div 10 \longrightarrow \div 100$

### Metric conversions

Useful Conversions

$\div 10$   $\div 100$   $\div 1000$

mm  $\longleftrightarrow$  cm  $\longleftrightarrow$  m  $\longleftrightarrow$  km

$\times 10$   $\times 100$   $\times 1000$

$\div 1000$

g  $\longleftrightarrow$  kg

$\times 1000$

$\div 1000$

ml  $\longleftrightarrow$  L

$\times 1000$

### Multiplication methods

Long multiplication (column)

Grid method

Repeated addition

Less effective method especially for bigger multiplication

#### Multiplication with decimals

Perform multiplications as integers  
e.g.  $0.2 \times 0.3 \longrightarrow 2 \times 3$

Make **adjustments** to your answer to match the question:  
 $0.2 \times 10 = 2$   
 $0.3 \times 10 = 3$

Therefore  $6 \div 100 = 0.06$

**Estimations:** Using estimations allows a 'check' if your answer is reasonable

### Division methods

Short division

$3584 \div 7 = 512$

$7 \overline{) 3584}$

Complex division

$\div 24 = \div 6 \div 4$   
Break up the divisor using factors

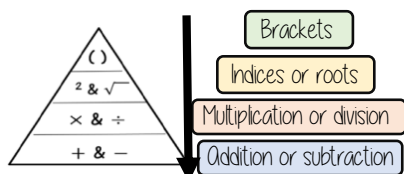
#### Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

$24 \div 0.02 \longrightarrow 24 \div 0.2 \longrightarrow 240 \div 2$

All give the same solution as represent the same proportion  
Multiply the values in proportion until the divisor becomes an integer

### Order of operations



If you have multiple operations from the same tier work from left to right

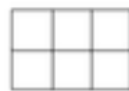
e.g.  $10 - 3 + 5 \longrightarrow 10 - 3 \longrightarrow 7 + 5$

$6 \times 4 + 8 \times 2$

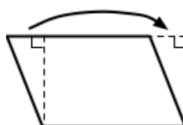
24 + 16 = 40

### Area problems

Rectangle  
Base x Perpendicular height

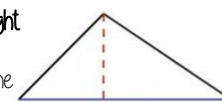


Parallelogram/ Rhombus  
Base x Perpendicular height



Triangle  
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$

A triangle is half the size of the rectangle it would fit in



### Mean problems

Mean – a measure of average  
It gives an idea of the central value

Lilly, Annie and Ezra have the following cubes

Lilly: 8 cubes  
Annie: 8 cubes  
Ezra: 8 cubes

24 in total

Finding the mean amount is the average amount each person would have if shared out equally

Lilly: 8 cubes  
Annie: 8 cubes  
Ezra: 8 cubes

The mean number of blocks would be 8 each

# YEAR 8 - DIRECTED NUMBER

## Operations with equations and directed numbers

### What do I need to be able to do?

By the end of this unit you should be able to:

- Perform calculations that cross zero
- Add/ Subtract directed numbers
- Multiply/ Divide directed numbers
- Evaluate algebraic expressions
- Solve two-step equations
- Use order of operations with directed number

### Keywords

**Subtract:** taking away one number from another.

**Negative:** a value less than zero

**Commutative:** changing the order of the operations does not change the result

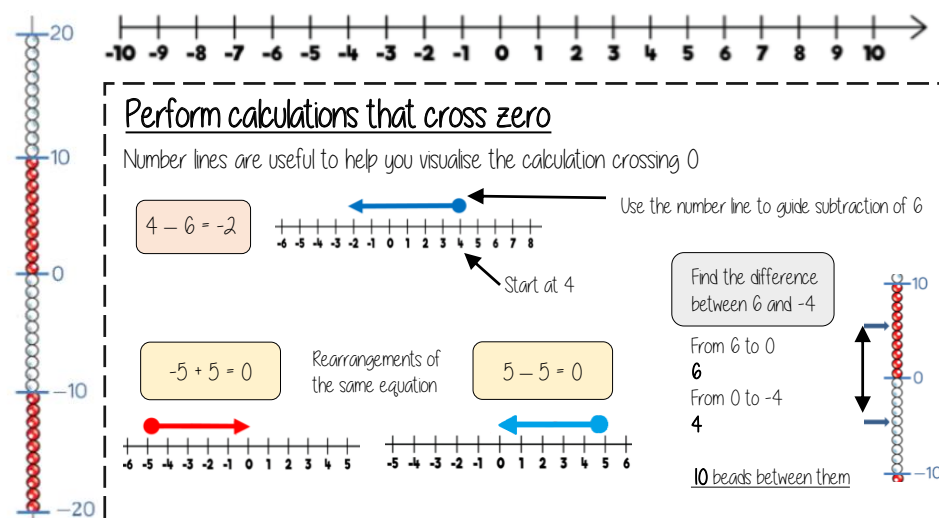
**Product:** multiply terms

**Inverse:** the opposite function

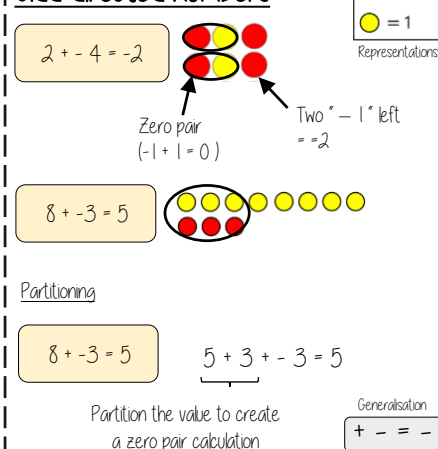
**Square root:** a square root of a number is a number when multiplied by itself gives the value (symbol  $\sqrt{\quad}$ )

**Square:** a term multiplied by itself.

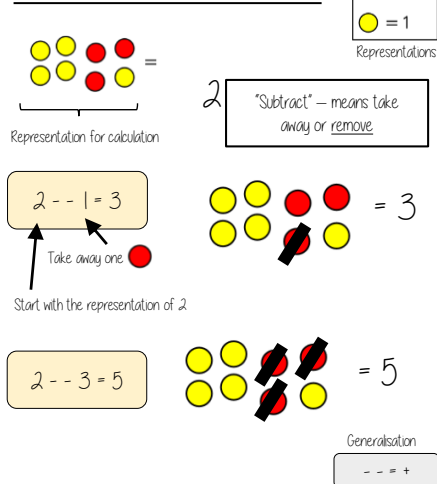
**Expression:** a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)



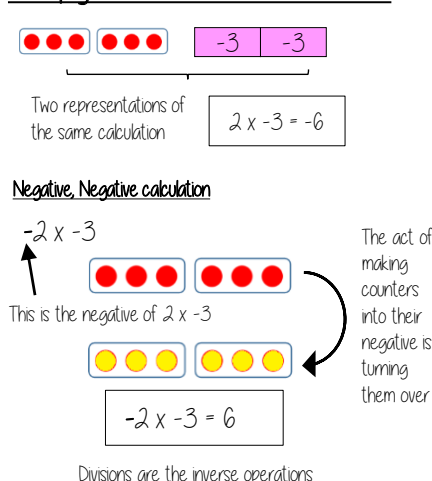
### Add directed numbers



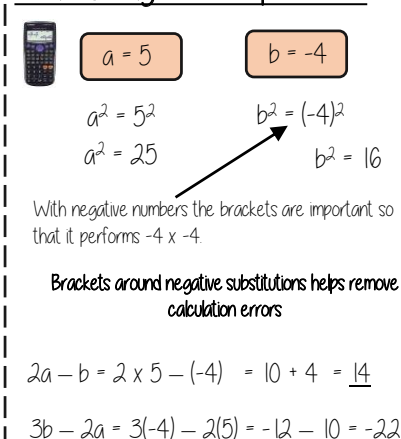
### Subtract directed numbers



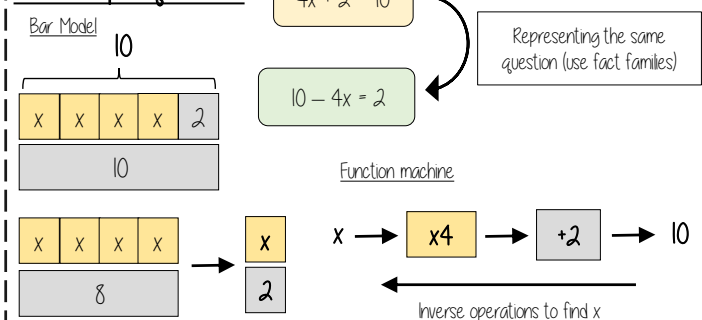
### Multiply/ Divide directed numbers



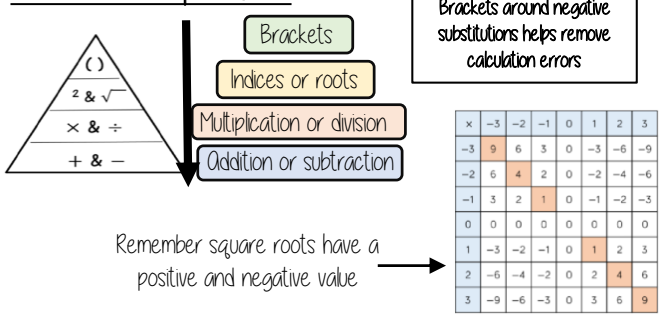
### Evaluate algebraic expressions



### Two-step equations



### Use order of operations



# YEAR 8 - PROPORTIONAL REASONING...

## Multiplying and Dividing Fractions

### What do I need to be able to do?

By the end of this unit you should be able to:

- Carry out any multiplication or division using fractions and integers
- Solutions can be modelled, described and reasoned

### Keywords

**Numerator**: the number above the line on a fraction. The top number. Represents how many parts are taken.

**Denominator**: the number below the line on a fraction. The number represents the total number of parts.

**Whole**: a positive number including zero without any decimal or fractional parts.

**Commutative**: an operation is commutative if changing the order does not change the result.

**Unit Fraction**: a fraction where the numerator is one and denominator a positive integer.

**Non-unit Fraction**: a fraction where the numerator is larger than one.

**Dividend**: the amount you want to divide up.

**Divisor**: the number that divides another number.

**Quotient**: the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

**Reciprocal**: a pair of numbers that multiply together to give 1.



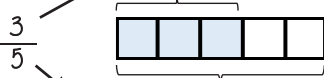
### Representing a fraction

**Numerator**

**Denominator**

Number of parts represented

**Numerator**

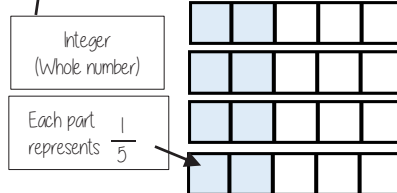


Number of parts to make up the whole  
**Denominator**

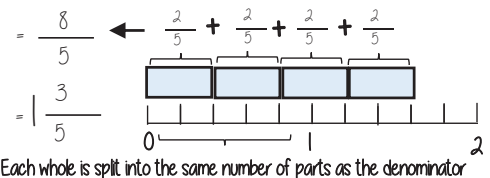
ALL PARTS of a fraction are of equal size

### Repeated addition = multiplication by an integer

$$4 \times \frac{2}{5} \rightarrow \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$$



How many parts are shaded?  
What each part represents



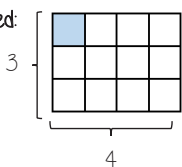
**Revisit**  
When adding fractions with the same denominator = add the numerators

### Multiplying unit fractions

$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

Parts shaded

Modelled:



Total number of parts in the diagram

### Multiplying non-unit fractions

Shade in 3 parts

Repeat it on this many rows

$$\frac{3}{4} \times \frac{2}{3}$$

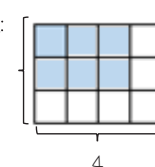
This many columns

This many rows

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Parts shaded

Modelled:



Total number of parts in the diagram

### Quick Multiplying and Cancelling down

$$\frac{1}{3} \times \frac{4}{9}$$

The 3 and the 9 have a common factor and can be simplified

Quick Solving

Multiply the numerators

Multiply the denominators

$$\frac{1 \times 4}{5 \times 3} = \frac{4}{15}$$

### The reciprocal

When you multiply a number by its reciprocal the answer is always 1

$$3 \times \frac{1}{3} = 1$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

The reciprocal of 3 is  $\frac{1}{3}$  and vice versa

**Reciprocals for division**

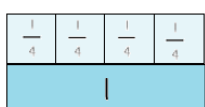
e.g.

$$5 \div \frac{1}{4} = 20$$

$$5 \times 4 = 20$$

Multiplying by a reciprocal gives the same outcome

### Dividing an integer by a unit fraction



$$1 \div \frac{1}{4} = 4$$

How many quarters are in 1?

"There are 4 quarters in 1 whole.

Therefore, there are 20 quarters in 5 wholes"

$$5 \div \frac{1}{4} = 20$$

### Dividing any fractions

Remember to use reciprocals

$$\frac{2}{5} \div \frac{3}{4}$$

$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

**Represented**



$$= \frac{8}{15}$$



# YEAR 8 - ALGEBRAIC TECHNIQUES...

## Brackets, Equations & Inequalities

### What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

### Keywords

**Simplify:** grouping and combining similar terms

**Substitute:** replace a variable with a numerical value

**Equivalent:** something of equal value

**Coefficient:** a number used to multiply a variable

**Product:** multiply terms

**Highest Common Factor (HCF):** the biggest factor (or number that multiplies to give a term)

**Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another

### Form expressions


For unknown variables, a letter is normally used in its place

More than – **ADD**

Less than/ difference – **SUBTRACT**

e.g 4 more than t  $\longrightarrow t + 4$   
 8 less than k  $\longrightarrow k - 8$

Only similar terms can be grouped together

t  e.g Find the perimeter of this shape  
 (Perimeter = length around outside of shape)  
 $2t + 1$   $t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$

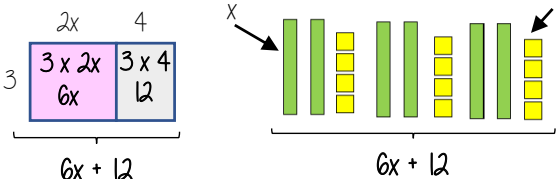
### Directed numbers

$++ \longrightarrow +$   
 $-- \longrightarrow +$   
 $+- \longrightarrow -$   
 $-+ \longrightarrow -$

e.g  $a = -5$  and  $b = 2$   
 $a^2 = a \times a = -5 \times -5 = 25$   
 $b + a = 2 + -5 = -3$

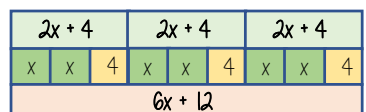
### Multiply single brackets

$3(2x + 4)$



$6x + 12$

Different representations of  $3(2x + 4) = 6x + 12$



### Factorise into a single bracket

$8x + 4$



Try and make this the highest common factor

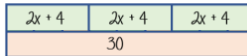
The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

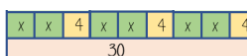
Note:  
 $8x + 4 \equiv 2(4x + 2)$   
 This is factorised but the HCF has not been used

### Solve equations with brackets

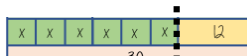
$3(2x + 4) = 30$



$6x + 12 = 30$



$6x = 18$



$x = 3$

$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12$

$6x = 18$

$-6$

Substitute to check your answer.  
 This could be negative or a fraction or decimal

$x = 3$

### Simple Inequalities

$<$  less than

$\leq$  Less than or equal to

$>$  More than

$\geq$  More than or equal to

$x < 10$

Say this out loud  
 "x is a value less than 10"

$10 > x$

Say this out loud  
 "10 is more than the value"

Note:  
 $x < 10$  and  $10 > x$   
 represent the same values

$x + 2 \leq 20$

"my value + 2 is less than or equal to 20"

$x \leq 18$

The biggest the value can be is 18

### Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

Form

$x \longrightarrow x \times 3 \longrightarrow +2 \longrightarrow 11$

$3x + 2 > 11$

Solve

$x \longleftarrow -3 \longleftarrow -2 \longleftarrow 11$

$x > 3$

Check

This would suggest any value bigger than 3 satisfies the statement

$3 \times 3 + 2 = 11 \checkmark$

$10 \times 3 + 2 = 32 \checkmark$

### Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Equation

A statement that two things are equal

Term

A single number or variable

Identity

An equation where both sides have variables that cause the same answer includes  $\equiv$

Formula

A rule written with all mathematical symbols  
 e.g area of a rectangle  $A = b \times h$

# YEAR 8 - REASONING WITH ALGEBRA...

## Forming and Solving Equations

### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

### Keywords

**Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another

**Variable:** a quantity that may change within the context of the problem

**Rearrange:** Change the order

**Inverse operation:** the operation that reverses the action

**Substitute:** replace a variable with a numerical value

**Solve:** find a numerical value that satisfies an equation

### Solve equations with brackets

R

$$3(2x + 4) = 30$$

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad -6$$

$$x = 3$$

$$x = 3$$

### Form and solve inequalities

R



Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

### Inequalities with negatives

Method 1 Make x positive first

$$2 - 3x > 17$$

$$+3x \quad +3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

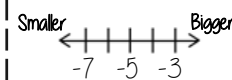
$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

**CHECK IT!**  
 $2 - 3(-6) = 20$   
 TRUE/ CORRECT



### Equations with unknown on both sides

$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$

$$x \quad x \quad x \quad x \quad 5$$

$$x \quad x \quad x \quad x \quad 24$$

### Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

Check it!

$$5(-8 + 4) < 3(-8 + 2)$$

$$5(-4) < 3(-6)$$

$$-20 < -18$$

✓ -20 IS smaller than -18

Method 2 Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

$$x < -5$$

When you multiply or divide x by a negative you need to reverse the inequality

### Formulae and Equations

Substitute in values

Formulae – all expressed in symbols

Equations – include numbers and can be solved

### Rearranging Formulae (one step)

$$x = y + z$$

$$x = y + z$$

Rearrange to make y the subject

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

### Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using  $y = mx + c$

e.g Find the gradient of the line  $2y - 4x = 9$

$$\text{Make y the subject first } y = \frac{4x + 9}{2}$$

$$\text{Gradient} = \frac{4}{2} = 2$$

# YEAR 8 - DEVELOPING GEOMETRY...

## Area of trapezia and Circles

### What do I need to be able to do?

By the end of this unit you should be able to:

- Recall area of basic 2D shapes
- Find the area of a trapezium
- Find the area of a circle
- Find the area of compound shapes
- Find the perimeter of compound shapes

### Keywords

**Congruent:** The same

**Area:** Space inside a 2D object

**Perimeter:** Length around the outside of a 2D object

**Pi ( $\pi$ ):** The ratio of a circle's circumference to its diameter.

**Perpendicular:** At an angle of  $90^\circ$  to a given surface

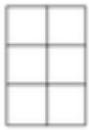
**Formula:** A mathematical relationship/ rule given in symbols. E.g  $b \times h$  = area of rectangle/ square

**Infinity ( $\infty$ ):** A number without a given ending (too great to count to the end of the number) — never ends

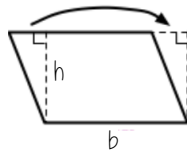
**Sector:** A part of the circle enclosed by two radii and an arc.

### Area — rectangles, triangles, parallelograms

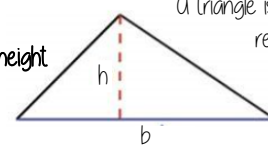
Rectangle  
Base x Height



Parallelogram/ Rhombus  
Base x Perpendicular height



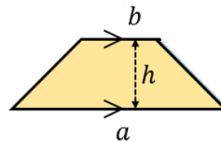
Triangle  
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$



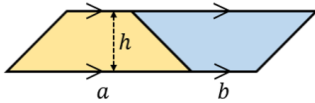
A triangle is half the size of the rectangle it would fit in

### Area of a trapezium

Area of a trapezium  
 $\frac{(a+b) \times h}{2}$



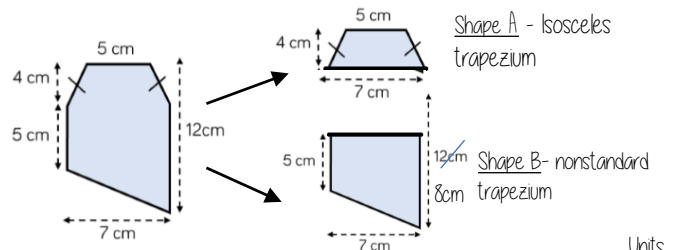
Why?



- Two congruent trapeziums make a parallelogram
- New length  $(a+b) \times \text{height}$
- Divide by 2 to find area of one

### Compound shapes

To find the area compound shapes often need splitting into more manageable shapes first. Identify the shapes and missing sides etc. first.



Shape A + Shape B = total area

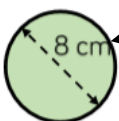
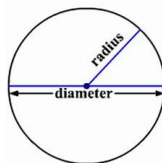
$$\frac{(5+7) \times 4}{2} + \frac{(5+8) \times 8}{2} = 24 + 45.5 = 69.5 \text{ cm}^2$$

Units

### Area of a circle (Non-Calculator)

Read the question — leave in terms of  $\pi$  or if  $\pi \approx 3$  (provides an estimate for answers)

Area of a circle  
 $\pi \times \text{radius}^2$



Diameter = 8cm  
 $\therefore$  Radius = 4cm

$$\begin{aligned} \pi \times \text{radius}^2 \\ = \pi \times 4^2 \\ = \pi \times 16 \\ = 16\pi \text{ cm}^2 \end{aligned}$$

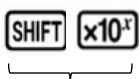
Find the area of one quarter of the circle



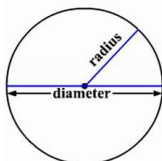
Radius = 4cm

$$\begin{aligned} \text{Circle Area} &= 16\pi \text{ cm}^2 \\ \text{Quarter} &= 4\pi \text{ cm}^2 \end{aligned}$$

### Area of a circle (Calculator)



Area of a circle  
 $\pi \times \text{radius}^2$



How to get  $\pi$  symbol on the calculator

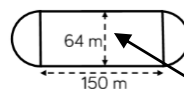
It is important to round your answer suitably — to significant figures or decimal places. This will give you a decimal solution that will go on forever!

### Compound shapes including circles

Circumference  
 $\pi \times \text{diameter}$

Compound shapes are not always area questions. For Perimeter you will need to use the circumference

Spotting diameters and radii



This dimension is also the diameter of the semi circles.

$$\begin{aligned} \text{Arc lengths} &= \pi \times 64 \\ &= 64\pi \end{aligned}$$

Don't need to halve this because there are 2 ends which make the whole circle

Arc lengths + Straight lengths = total perimeter

$$\begin{aligned} &= 64\pi + 150 + 150 \\ &= (300 + 64\pi) \text{ m} \\ \text{OR } &= 501.1 \text{ m} \end{aligned}$$

Still remember to split up the compound shape into smaller more manageable individual shapes first

# YEAR 8 - CONSTRUCTING IN 2D/3

## 3D Shapes

### What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

### Keywords

**2D:** two dimensions to the shape e.g. length and width

**3D:** three dimensions to the shape e.g. length, width and height

**Vertex:** a point where two or more line segments meet

**Edge:** a line on the boundary joining two vertex

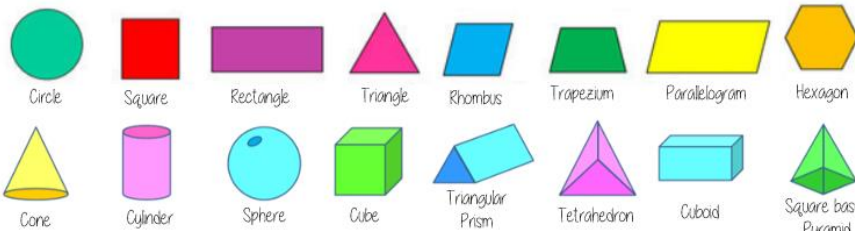
**Face:** a flat surface on a solid object

**Cross-section:** a view inside a solid shape made by cutting through it

**Plan:** a drawing of something when drawn from above (sometimes birds eye view)

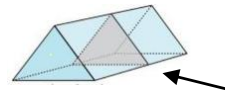
**Perspective:** a way to give illustration of a 3D shape when drawn on a flat surface.

### Name 2D & 3D shapes



### Recognise prisms

A solid object with two identical ends and flat sides

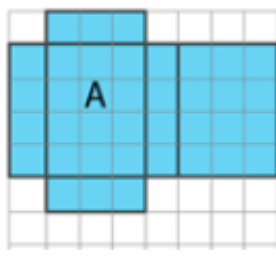
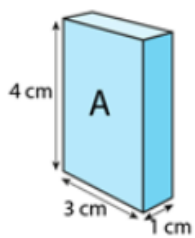


The cross section will also be identical to the end faces



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

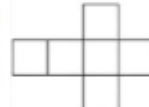
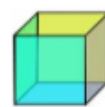
### Nets of cuboids



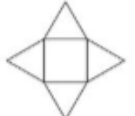
1cm grids help to draw accurately

Visualise the folding of the net  
Will it make the cuboid with all sides touching

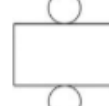
### Sketch and recognise nets



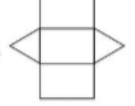
Do they have the same number of faces?



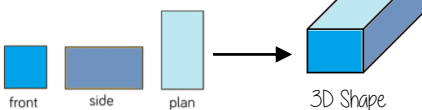
Where do the edges join?



Are the shapes of the faces correct?



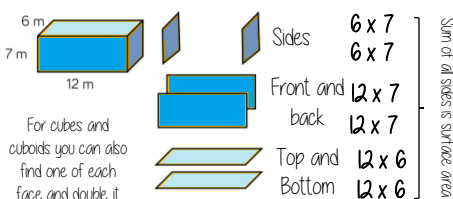
### Plans and elevations



The direction you are considering the shape from determines the front and side views

### Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



For cubes and cuboids you can also find one of each face and double it

Sum of all sides is surface area



For other shapes - not all the sides are the same, so calculate the individually

### Area of 2D shapes

Rectangle  
Base x Height



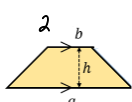
Triangle  
 $\frac{1}{2} \times \text{Base} \times \text{Perpendicular height}$



Parallelogram/ Rhombus  
Base x Perpendicular height



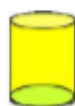
Area of a trapezium  
 $\frac{(a+b) \times h}{2}$



Area of a circle  
 $\pi \times \text{radius}^2$



### Surface area - cylinders



The area of the circle  
 $\pi \times \text{radius}^2$

The width of this face is the same as the circumference  
 $\pi \times \text{diameter} \times \text{height}$

$$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$$

### Volumes

Volume is the 3D space it takes up - also known as capacity if using liquids to fill the space



#### Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

$$\text{Cubes/ Cuboids} = \text{base} \times \text{width} \times \text{height}$$

Remember multiplication is commutative



Cross section



Cross section

$$\text{Prisms and cylinders} = \text{area cross section} \times \text{height}$$

Height can also be described as depth

Areas - square units  
Volumes - cube units

Areas and volumes can be left in terms of  $\pi$





# YEAR 8 - REASONING WITH GEOMETRY...

## Solving ratio & proportion problems

### What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

### Keywords

**Proportion:** a comparison between two numbers

**Ratio:** a ratio shows the relative size of two variables

**Direct proportion:** as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

**Inverse proportion:** as one variable is multiplied by a scale factor the other is divided by the same scale factor.

### Direct Proportion

As one variable changes the other changes at the same rate.

**R**



4 cans of pop = £2.40

4 cans of pop = £2.40  
2 cans of pop = £1.20

This multiplier is the same  
In the same way that this  
would be for ratio

This is a multiplicative change

4 cans of pop = £2.40  
12 cans of pop = £7.20

Sometimes this is easiest  
if you work out how much  
one unit is worth first  
e.g. 1 can of pop = £0.60

### Conversion Graphs

Compare two variables

**R**



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare — then find the associated point by using your graph.

Using a ruler helps for accuracy  
Showing your conversion lines help as a "check" for solutions

### Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

Arrows indicate: 1 to 2 is  $\times 2$ , 2 to 8 is  $\times 4$ , 40 to 20 is  $\div 2$ , 20 to 5 is  $\div 4$ .

### Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts



**Shop A**

4 cans for £1.20

£1.20 ÷ 4

Cost per item

1 can is £0.30  
Or 30p

**Shop B**

3 cans for 93p

£0.93 ÷ 3

1 can is £0.31  
Or 31p

Shop A is the best value as it is 1p cheaper per can of pop



**Shop A**

4 cans for £1.20

4 ÷ £1.20

Cost per pound

£1 buys 3.333 cans of pop

3 cans for 93p

3 ÷ £0.93

£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound.

Best value is the most product for the lowest price per unit

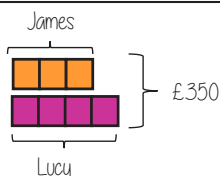
### Sharing a whole into a given ratio

**R**

James and Lucy share £350 in the ratio 3:4  
Work out how much each person earns

Model the Question

James: Lucy  
3 : 4



£350 ÷ 7 = £50

□ = one part  
= £50

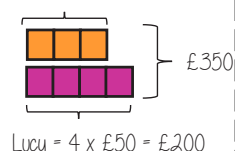
Find the value of one part

Whole: £350  
7 parts to share between  
(3 James, 4 Lucy)

Put back into the question

James: Lucy  
(x 50) 3 : 4 (x 50)  
£150 : £200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

### Finding a value given 1:n (or n:1)

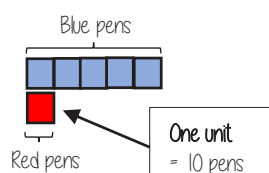
**R**

Inside a box are blue and red pens in the ratio 5:1  
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red  
5 : 1

□ = one part  
= 10 pens

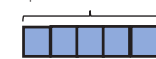


Put back into the question

Blue: Red

(x 10) 5 : 1 (x 10)  
50 : 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

# YEAR 8 - REASONING WITH DATA...

## The data handling cycle

### What do I need to be able to do?

By the end of this unit you should be able to:

- Set up a statistical enquiry
- Design and criticise questionnaires
- Draw and interpret multiple bar charts
- Draw and interpret line graphs
- Represent and interpret grouped quantitative data
- Find and interpret the range
- Compare distributions

### Keywords

**Hypothesis:** an idea or question you want to test

**Sampling:** the group of things you want to use to check your hypothesis

**Primary Data:** data you collect yourself

**Secondary Data:** data you source from elsewhere e.g. the internet/ newspapers/ local statistics

**Discrete Data:** numerical data that can only take set values

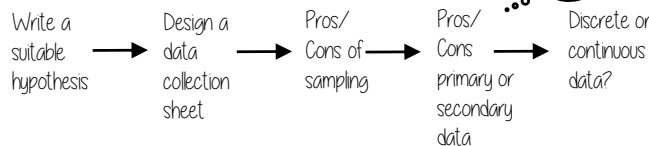
**Continuous Data:** numerical data that has an infinite number of values (often seen with height, distance, time)

**Spread:** the distance/ how spread out/ variation of data

**Average:** a measure of central tendency – or the typical value of all the data together

**Proportion:** numerical relationship that compares two things

### Set up a statistical enquiry



Features of a data collection sheet

	Data Title	Tally	Frequency
Grouped or ungrouped categories			

Total number of that group observed

### Design and criticise a questionnaire

**The Question** – be clear with the question – don't be too leading/ judgemental

e.g. How much pocket money do you get a week?

**Responses** – do you want closed or open responses? – do any options overlap? – Have you an option for all responses?

Zero option → ☐ £0 ☐ £0.01 - £2 ☐ £2.01 - £4 ☐ more than £4 ← More option

NOTE: For responses about continuous data include inequalities  $< x \leq$

### Pictograms, bar and line charts

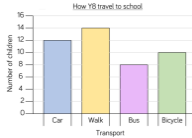
**Pictogram**

Language	
French	●●●●●
Spanish	●●●●●
German	●●●●●

● - 4 people

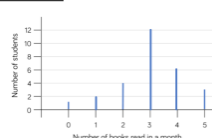
- Need to remember a key
- Visually able to identify mode

**Bar Chart**



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

**Line Chart**



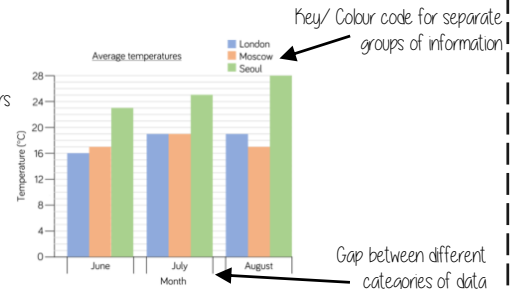
- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

Represents quantitative data

### Multiple Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other



### Draw and interpret Pie Charts

Remember a circle has 360°

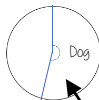
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$  "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$$\frac{32}{60} \times 360 = 192^\circ$$



Use a protractor to draw This is 192°

**Multiple method**

As 60 goes into 360 – 6 times  
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

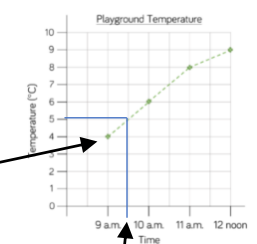
Represents quantitative, discrete data

### Draw and interpret line graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points

Line graphs do not need to start from 0

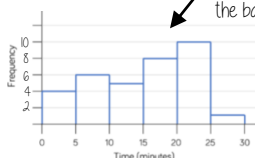
More than one piece of data can be plotted on the same graph to compare data



It is possible to make estimates from the line e.g. temperature at 9.30am is 5°C

### Grouped quantitative data

Time (minutes)	Frequency
$0 \leq t < 5$	4
$5 \leq t < 10$	6
$10 \leq t < 15$	5
$15 \leq t < 20$	8
$20 \leq t < 25$	10
$25 \leq t < 30$	1



This is a frequency diagram There are no gaps between the bars

Grouping the data is useful if there is a large spread of data to begin with

"More than or equal to 25 and less than 30 minutes"

The use of inequalities shows that this will be a frequency diagram

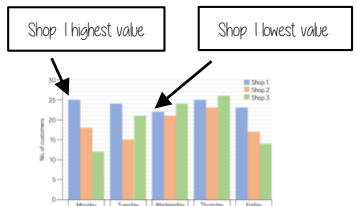
### Find and interpret the range

The range is a measure of spread

A smaller range means there is less variation in the results – it is more consistent data

A range of 0 means all the data is the same value

Difference between the biggest and smallest values



Range of customers =  $25 - 22 = 3$  (Shop 1)

Shop 1 has the smallest range – this indicates it has a more consistent flow of customers each week

# YEAR 8 - REASONING WITH GEOMETRY...

## Pythagoras' theorem

### What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

### Keywords

**Square number:** the output of a number multiplied by itself

**Square root:** a value that can be multiplied by itself to give a square number

**Hypotenuse:** the largest side on a right angled triangle. Always opposite the right angle.

**Opposite:** the side opposite the angle of interest

**Adjacent:** the side next to the angle of interest

### Squares and square roots



This can also be written as  $6^2$

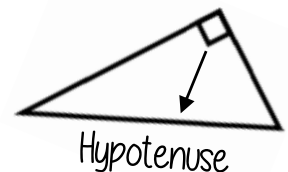
$\sqrt{\quad}$  is the square root symbol

e.g.  $\sqrt{64} = 8$   
Because  $8 \times 8 = 64$

1 x 1	2 x 2	3 x 3	4 x 4	5 x 5	6 x 6	7 x 7	8 x 8	9 x 9	10 x 10
1	4	9	16	25	36	49	64	81	100

Square numbers

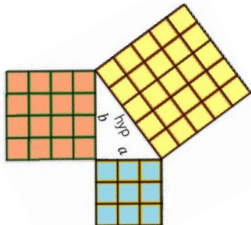
### Identify the hypotenuse



Hypotenuse

The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.

### Determine if a triangle is right-angled



If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

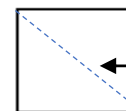
e.g.  $a^2 + b^2 = \text{hypotenuse}^2$

$$3^2 + 4^2 = 5^2$$

$$9 + 16 = 25$$

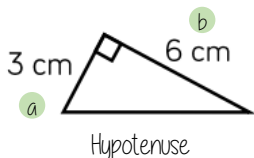
Substituting the numbers into the theorem shows that this is a right-angled triangle

$a = 3$   $b = 4$   $c = 5$



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

### Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

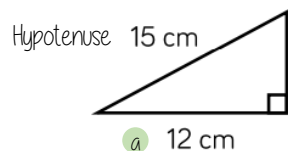
$$45 = \text{hypotenuse}^2$$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

$$\sqrt{45} = \text{hypotenuse}$$

$$6.71\text{cm} = \text{hypotenuse}$$

### Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1 Substitute in the values you are given

$$144 + b^2 = 225$$

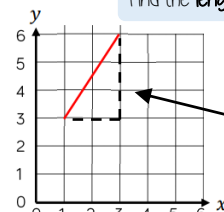
$$-144 \quad -144$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

$$\begin{aligned} \text{Square root to find the length of the side} \quad & \left\{ \begin{aligned} b^2 &= 111 \\ b &= \sqrt{111} = 10.54 \text{ cm} \end{aligned} \right. \end{aligned}$$

### Pythagoras' theorem on a coordinate axis

Find the length of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram

The line segment is the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes