YEAR 9 - DEVELOPING NUMBER...
Standard Form

What do I need to be able to do?

By the end of this unit you should be able to:

- Write numbers in standard form and as ordinary numbers
- Order numbers in standard form
- Add/ Subtract with standard from
- Multiply/ Divide with standard form
- Use a calculator with standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent — or the number that tells you how many times to use the number in multiplication
Exponent: The power — or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.

Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

$A \times 10^n$

Any number between 1 and less than 10 — A; Any integer — n

Example
3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example
0.8×10^4
$5.3 \times 10^{0.7}$

Negative powers of 10

0.001
$1 \times \frac{1}{1000}$
1×10^{-3}

10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	•	10^{-1}	10^{-2}	10^{-3}
0	0	•	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	•	10^{-1}	10^{-2}	10^{-3}
0	•	0	5	4

A negative power does not mean a negative answer — it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	•	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= 6.4 \times 10^5$

$8 \times 10^5 \times 3$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= 2.4 \times 10^6$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= 0.5 \times 10^3$

Remember the layout for standard form

$A \times 10^n$

Any number between 1 and less than 10 — A; Any integer — n

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$6 \times 10^5 + 8 \times 10^5$

Method 1
$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^5$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2
$= (6 + 8) \times 10^5$
$= 14 \times 10^5$ This is not the final answer
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^5$

Only works if the powers are the same

Multiplication and division

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$ Division questions can look like this

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \times 10^5 \div 10^3$

$= 5 \times 10^2$

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

Using a calculator

$14 \times 10^5 \times 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy

Input 14 and press **x10ˣ** Then press 5 (for the power)
Press **×**
Input 3.9 and press **x10ˣ** Then press 3 (for the power)
Press **=**

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press **SHIFT** **SETUP** and then press 7 for sci mode

Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

YEAR 9 - REASONING WITH NUMBER...

Numbers

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify integers, real and rational numbers
- Work with directed number
- Solve problems with number
- Find HCF/ LCM
- Add/ Subtract fractions
- Multiply/ Divide fractions
- Write numbers in standard form

Keywords

Integer: a whole number that is positive or negative
Rational: a number that can be made by dividing two integers
Irrational: a number that cannot be made by dividing two integers
Inverse operation: the operation that reverses the action
Quotient: the result of a division
Product: the result of a multiplication
Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2:1 ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat

The square root of a negative is not a real number and cannot be found

HCF/LCM R

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9	9, 18, 27, 36, 45, 54
12	12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Standard form R

Any number between 1 and less than 10 → $A \times 10^n$ ← Any integer

$6 \times 10^5 + 8 \times 10^5$

$= 600000 + 800000$

$= 1400000$

$= 1.4 \times 10^5$

$(1.5 \times 10^5) \div (0.3 \times 10^3)$

$1.5 \div 0.3 \quad \times \quad 10^5 \div 10^3$

$= 5 \times 10^2$

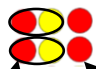
Directed number R

= –1
= 1

Addition

Generalisation: $+ - = -$

$2 + -4 = -2$

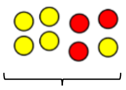


Zero pair (-1 + 1 = 0)

Two "– 1" left = -2

Subtraction

Generalisation: $- - = +$


= 2

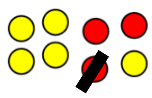
Representation for calculation

"Subtract" – means take away or remove

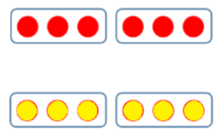
$2 - -1 = 3$

Take away one

Start with the representation of 2


= 3

Multiplication



The act of making counters into their negative is turning them over

$-2 \times -3 = 6$

Divisions are the inverse operations

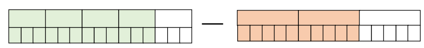
a = 5 b = -4

Brackets around negative substitutions helps remove calculation errors

$2a - b = 2 \times 5 - (-4) = 10 + 4 = \underline{14}$

Addition/ Subtraction of fractions R

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

Multiplication/ Division of fractions R

Shade in 3 parts → $\frac{3}{4} \times \frac{2}{3}$ ← Repeat it on this many rows

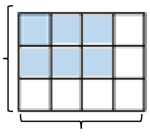
This many columns ↑ ↑ This many rows

$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$

Parts shaded

Total number of parts in the diagram

Modelled:



3

4

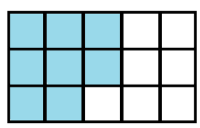
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

YEAR 9 - REASONING WITH NUMBER...
Using Percentages

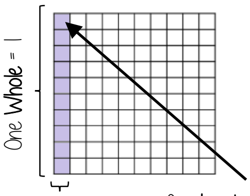
What do I need to be able to do?

By the end of this unit you should be able to:
- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

Percent: parts per 100 – written using the % symbol
Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
Fraction: a fraction represents how many parts of a whole value you have.
Equivalent: of equal value.
Reduce: to make smaller in value.
Growth: to increase/ to grow.
Integer: whole number, can be positive, negative or zero.
Invest: use money with the goal of it increasing in value over time (usually in a bank).
Multiplier: the number you are multiplying by.
Profit: the income take away any expenses/ costs.

FDP Equivalence R



Percentage
100% = a whole = 100 hundredths

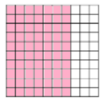
10 hundredths
10 out of 100
10%

$\frac{10}{100} = \frac{1}{10} = 0.10$ One hundredth (one whole split into 100 equal parts)

ones	tenths	hundredths
•	•	

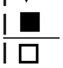
Converting FDP R

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator



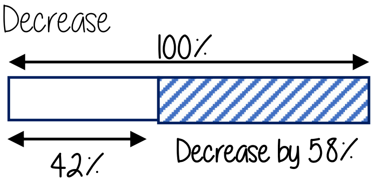
→ S⇔D Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333
= $0.\dot{3}$
The dot above the 3

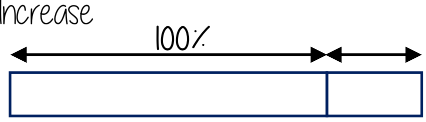
Percentage Increase/ Decrease R

Decrease



1.00 – 0.58 = 0.42 ← Multiplier Less than 1

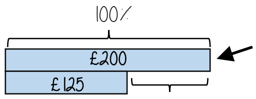
Increase



100% + 12% = 112%
1.00 + 0.12 = 1.12 ← Multiplier More than 1

Percentage change R

I bought a phone for £200. A year later sold it for £125.



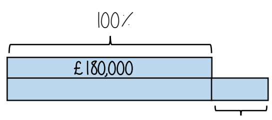
All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

$$\frac{\textit{Difference in values}}{\textit{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000.

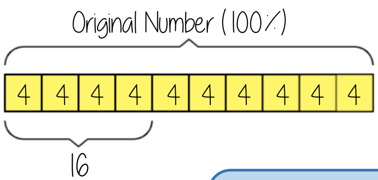


Percentage profit

Money made (profit value) → $\frac{36000}{180000} \times 100 = 20\%$

Reverse Percentages

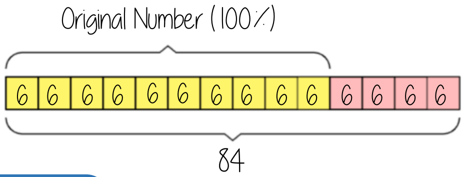
40% of my number is 16. What am I thinking of?



40% = 16
10% - 4
100% = 40

Try to scale down to 10% or 1% and then scale back up to 100%

140% of my number is 84. What is the original number?



140% = 84
10% - 6
100% = 60

YEAR 9 - REASONING WITH NUMBER...
Maths & Money

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

Keywords

Credit: money being placed into a bank account
Debit: money that leaves a bank account
Balance: the amount of money in a bank account
Expense: a cost/ outgoing
Deposit: an initial payment (often a way of securing an item you will later pay for)
Multiplier: a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)
Per Annum: each year
Currency: the type of money a country uses.
Unitary: one – the cost of one.

Bills and Bank Statements

Bills – tell you the amount items cost and can show how much money you need to pay

Some can include a total

Look for different units
(Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Setp	Bday Money	£15		£915

Simple Interest

For each year of investment the interest remains the same

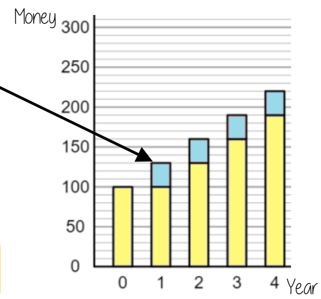
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account

eg Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned **£120** interest.
At the end of year 4 they have **£220**



Compound Interest

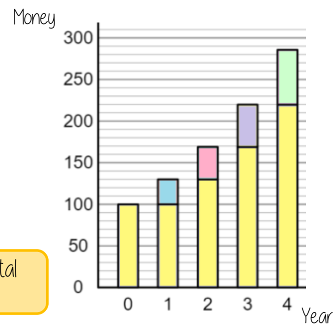
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

eg Invest £100 at 30% compound interest for 4 years

$100 \times 1.3^4 = £285.61$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

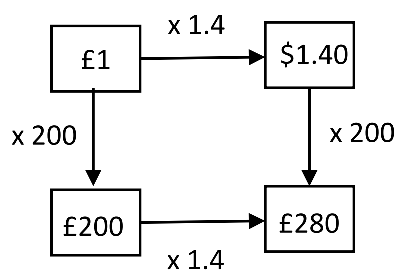
Wages and Taxes

Salaries fall into tax brackets – which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:
Time and a half – means 1.5 times their hourly rate
Double – 2 times their hourly rate

Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies		
United Kingdom	£	Pounds
United States of America	$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges	5 cupcakes
£1	£1.20

$4 = £1.00$ ÷ 2
$2 = £0.50$ ÷ 2
$1 = £0.25$

$5 = £1.20$ ÷ 5
$1 = £0.20$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

YEAR 9 - DEVELOPING NUMBER...

Number Sense

What do I need to be able to do?

By the end of this unit you should be able to:
- Round numbers to powers of 10 and 1 sf
- Round numbers to any dp
- Estimate solutions
- Calculate using order of operations
- Calculate with money, units of measurement and time

Keywords

Significant: Place value of importance
Round: Making a number simpler but keeping its value close to what it was.
Decimal: Place holders after the decimal point.
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value.
Metric: A system of measurement.
Balance: The amount of money in a bank account
Deposit: Putting money into a bank account.

Round to powers of 10 and 1 sig figure

R

If the number is halfway between we "**round up**"

5495 to the nearest 1000

5000 ↑ 6000

5475 to the nearest 100

5400 ↑ 5500

5475 to the nearest 10

5470 ↑ 5480

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

Round to decimal places

2.46192 ← Focus on the numbers **after** the decimal point

"To 1dp" – to one number after the decimal
"To 2dp" – to two numbers after the decimal

2.46192 (to 1dp) - Is this closer to 2.4 or 2.5

2.4 ↑ 2.5

2.4 | 6192 This shows the number is closer to 2.5

2.46192 (to 12dp) - Is this closer to 2.46 or 2.47

2.46 ↑ 2.47

2.46 | 192 This shows the number is closer to 2.46

Estimate the calculation

Round to 1 significant figure to estimate

$4.2 + 6.7 \approx 4 + 7 \approx 11$ This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

$21.4 \times 3.1 \approx 20 \times 3 \approx 60$ This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Order of operations

R

Brackets Operations in brackets are calculated first
Other operations e.g. powers, roots,
Multiplication/ Division
They are carried out in the order from left to right in the question
Addition/ Subtraction
They are carried out in the order from left to right in the question

Calculations with money

Debit - You have £0 or more in an account

Credit - You have less than £0 in an account

Money calculations are to 2dp

Using a calculator – ensure you are working in the correct units.
£1.30 + 50p = 130 + 50 (in pence)
= 1.30 + 0.50 (in pounds)

£1 = 100p

Units are important: Useful Conversions

mm $\xrightarrow{\div 10}$ cm $\xrightarrow{\div 100}$ m $\xrightarrow{\div 1000}$ km; km $\xrightarrow{\times 1000}$ m $\xrightarrow{\times 100}$ cm $\xrightarrow{\times 10}$ mm

g $\xrightarrow{\div 1000}$ kg; kg $\xrightarrow{\times 1000}$ g

ml $\xrightarrow{\div 1000}$ L; L $\xrightarrow{\times 1000}$ ml

Metric measures of length

Kilo = 1000 x meter
Centi - $\frac{1}{100}$ x meter
Milli - $\frac{1}{1000}$ x meter

Units of weight/ capacity

Weight = g, kg, t
Capacity (volume of liquid) = ml, L

Time and the calendar

1 Year – the amount of time it takes Earth to go around the sun 365 (and a quarter) **days**
Leap Year – 366 **days** (every 4 years)

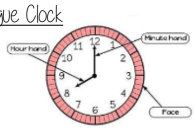
12 Months = one year = 52 weeks
31 days – **Jan, March, May, July Aug, Oct, Dec**
30 days – **April, June, Sept, Nov**
28 days – **Feb** (29 leap year)

1 week – 7 days
Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday

1 day – 24 hours
1 hour - 60 minutes
1 minute – 60 seconds

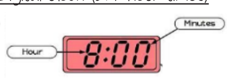
Use a number line for time calculations!

Analogue Clock



12-hour clock
- Use am (morning) and pm (afternoon)
- Only use hour times up to 12

Digital Clock (24-hour times)



24-hour clock
- 0-11 (morning hours)
- 12-23 (afternoon hours)

YEAR 9 - REASONING WITH GEOMETRY...

Solving ratio & proportion problems

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers
Ratio: a ratio shows the relative size of two variables
Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.
Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

R

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40
× 0.5 2 cans of pop = £1.20 × 0.5

This multiplier is the same
In the same way that this would be for ratio

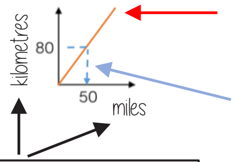
4 cans of pop = £2.40
× 3 12 cans of pop = £7.20 × 3

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables

R



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2 × 4

T	1	2	8
G	40	20	5

× 2 ÷ 4

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

	Shop A	Shop B
	4 cans for £1.20	3 cans for 93p
	£1.20 ÷ 4	£0.93 ÷ 3
Cost per item	1 can is £0.30 Or 30p	1 can is £0.31 Or 31p

Shop A is the best value as it is 1p cheaper per can of pop

	Shop A	
	4 cans for £1.20	3 cans for 93p
	4 ÷ £1.20	3 ÷ £0.93
Cost per pound	£1 buys 3.333 cans of pop	£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

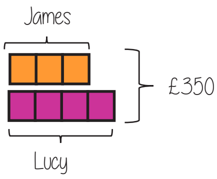
Sharing a whole into a given ratio

R

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

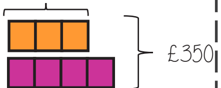
£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James: Lucy
× 50 3 : 4 × 50
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Finding a value given 1:n (or n:1)

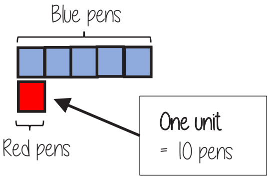
R

Inside a box are blue and red pens in the ratio 5:1.
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

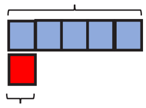
□ = one part = 10 pens



Put back into the question

Blue : Red
× 10 5 : 1 × 10
50: 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

YEAR 9 - REASONING WITH GEOMETRY...

Rates

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

Keywords

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight.

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

Substitute: putting numbers where letters are – replacing numbers into a formula

Speed, Distance, Time

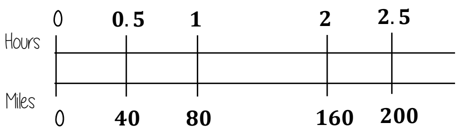
"per" for every

e.g. 80 miles per hour (mph)

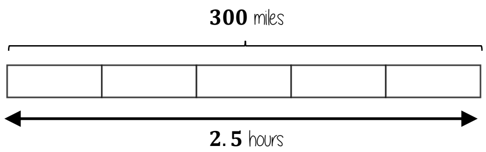
Travel 80 miles every hour

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.



Bar models can help to calculate mph

Each part is half an hour

Each part is 60 miles

Speed, Distance, Time

Before calculations – make sure you are working in the same units as the speed

minutes $\xrightarrow{\div 60}$ hours

hours $\xrightarrow{\times 60}$ minutes

Learn or learn how to rearrange the formular for speed, distance and time

Substitute in the variables given

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

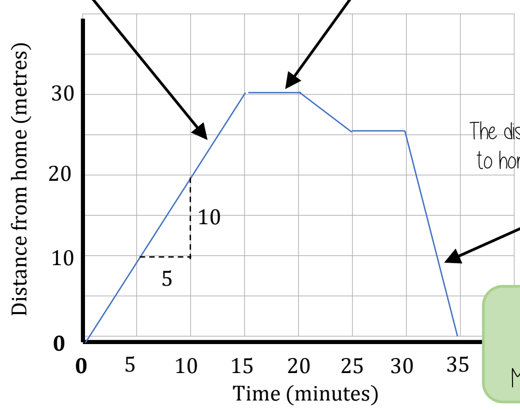
Distance – Time graphs

Gradient = speed

The steeper a gradient the faster the speed

$\frac{10}{5} = 2$ metres per min

Horizontal lines represent staying still



The distance coming closer to home shows the return journey

Units are important

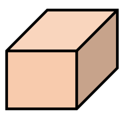
Meters per minute

Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

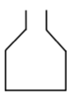
$$\text{mass} = \text{volume} \times \text{density}$$



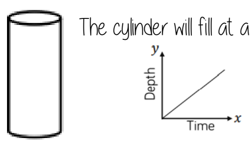
volume of prism = Area of cross section × Depth

R

Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed

Units are important

Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles **per** hour

Exchange rates: euros **per** pounds

Density: mass **per** volume

metres $\xrightarrow{\div 1000}$ kilometres

kilometres $\xrightarrow{\times 1000}$ metres

R

YEAR 9 – REASONING WITH ALGEBRA…
Forming and Solving Equations

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another
Variable: a quantity that may change within the context of the problem
Rearrange: Change the order
Inverse operation: the operation that reverses the action
Substitute: replace a variable with a numerical value
Solve: find a numerical value that satisfies an equation

Solve equations with brackets

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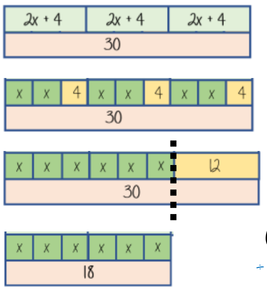


$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$
$-12 \quad -12$
$6x = 18$
$\div 6 \quad \div 6$
$x = 3$

Form and solve inequalities

R

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$
$+3x \quad +3x$
$2 > 17 + 3x$
$-17 \quad -17$
$-15 > 3x$
$\div 3 \quad \div 3$
$-5 > x$

x is true for any value smaller than -5

Smaller Bigger
-7 -5 -3

CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Method 2 Keep the negative x

$2 - 3x > 17$
$-2 \quad -2$
$-3x > 15$
$\div -3 \quad \div -3$
$x > -5$

x is true for any value bigger than -5

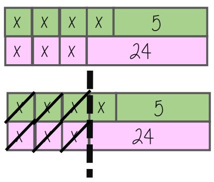
This cannot be true…

$x < -5$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

$4x + 5 = 3x + 24$
$-3x \quad -3x$
$x + 5 = 24$
$-5 \quad -5$
$x = 19$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$
$5x + 20 < 3x + 6$
$2x + 20 < 6$
$2x < -14$
$x < -7$

Check it!

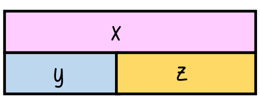
$5(-8 + 4) < 3(-8 + 2)$
$5(-4) < 3(-6)$
$-20 < -18$
-20 IS smaller than -18

Formulae and Equations

Substitute in values

Formulae – all expressed in symbols → **Equations** – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$
Rearrange to make y the subject.
$y = x - z$

$y \xrightarrow{+z} x$
$y \xleftarrow{-z} x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$
$+3 \quad +3$
$4x = 12$
$\div 4 \quad \div 4$
$x = 3$

In a formula (make x the subject)

$xy - s = a$
$+s \quad +s$
$xy = a + s$
$\div y \quad \div y$
$x = \frac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

eg Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$ Gradient $= \frac{4}{2} = 2$

YEAR 9 - REASONING WITH ALGEBRA...
Straight Line Graphs

What do I need to be able to do?

By the end of this unit you should be able to:

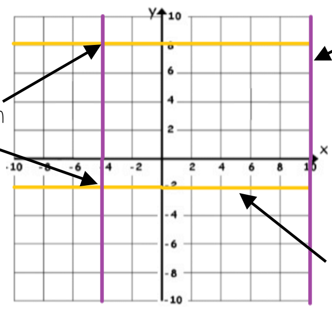
- Compare gradients
- Compare intercepts
- Understand and use y= mx + c
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line
Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.
Parallel: two lines that never meet with the same gradient.
Co-ordinate: a set of values that show an exact position on a graph.
Linear: linear graphs (straight line) – linear common difference by addition/ subtraction
Asymptote: a straight line that a graph will never meet.
Reciprocal: a pair of numbers that multiply together to give 1.
Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes

R



All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Plotting y = mx + c graphs

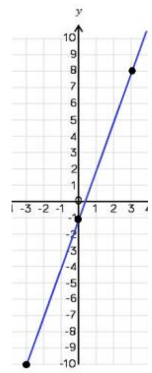
R

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

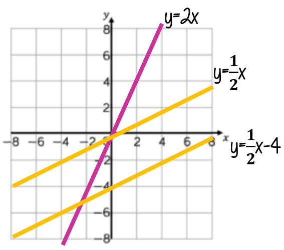
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient

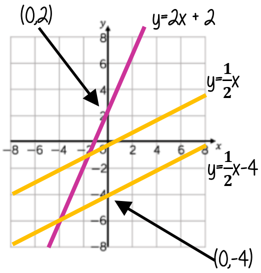
Positive gradients

Negative gradients

Compare Intercepts

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

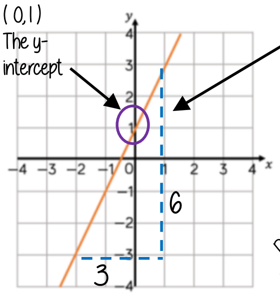
The equation of a line can be rearranged. Eg:

y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient
$\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

YEAR 9 - REPRESENTATIONS...

Algebraic Representation

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

Keywords

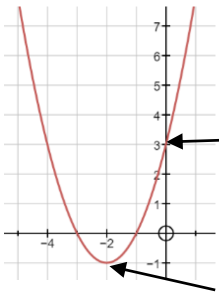
Quadratic: a curved graph with the highest power being 2. Square power.
Inequality: makes a non equal comparison between two numbers
Reciprocal: a reciprocal is 1 divided by the number
Cubic: a curved graph with the highest power being 3. Cubic power.
Origin: the coordinate (0, 0)
Parabola: a 'U' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Intersection with the y axis

Quadratic graphs are always symmetrical with the turning point in the middle

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

Plot all of the coordinate pairs and join the points with a curve (freehand)

Interpret other graphs

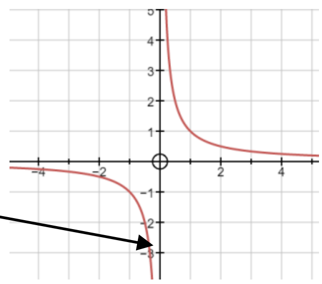
Cubic Graphs

$y = x^3 + 2x^2 - 2x + 1$

If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

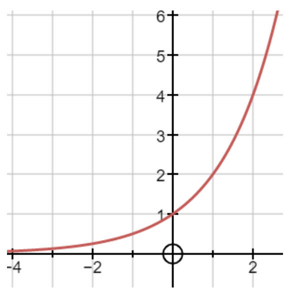


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x

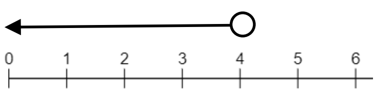


Represent Inequalities

Multiple methods of representing inequalities

$x < 4$

All values are less than 4

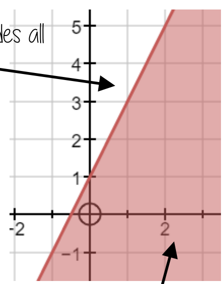


The shaded area indicates all possible values of x

The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$y \geq 2x + 1$



The shaded area indicates all possible solutions to this inequality

YEAR 9 - DEVELOPING GEOMETRY...
Line symmetry and reflection

What do I need to be able to do?

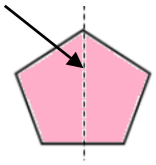
By the end of this unit you should be able to:
- Recognise line symmetry
- Reflect in a horizontal line
- Reflect in a vertical line
- Reflect in a diagonal line

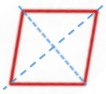
Keywords

Mirror line: a line that passes through the center of a shape with a mirror image on either side of the line
Line of symmetry: same definition as the mirror line
Reflect: mapping of one object from one position to another of equal distance from a given line.
Vertex: a point where two or more-line segments meet.
Perpendicular: lines that cross at 90°
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)

Lines of symmetry

Mirror line (line of reflection)



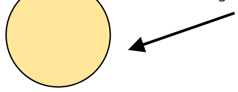


Rhombus
two lines of symmetry

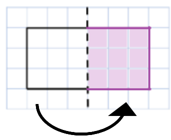
Parallelogram
No lines of symmetry

Shapes can have more than one line of symmetry…
This regular polygon (a regular pentagon has 5 lines of symmetry)

A circle has an infinite amount of lines of symmetry

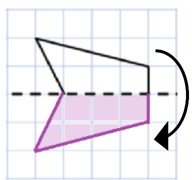


Reflect horizontally/ vertically (1)



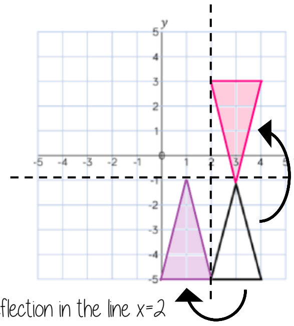
Note: a reflection doubles the area of the original shape

Reflection in a vertical line



Reflection in a horizontal line

Reflection on an axis grid

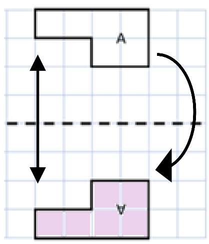


Reflection in the line y=-2

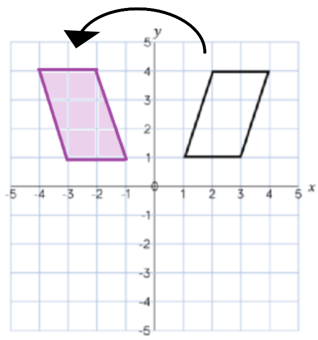
Reflection in the line x=2

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



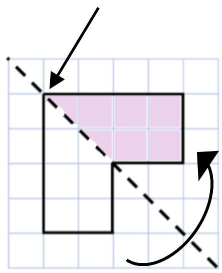
Reflection in the line y axis – this is also a reflection in the line x=0



Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

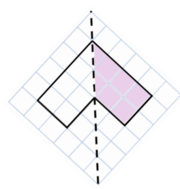
Reflect Diagonally (1)

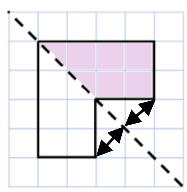
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

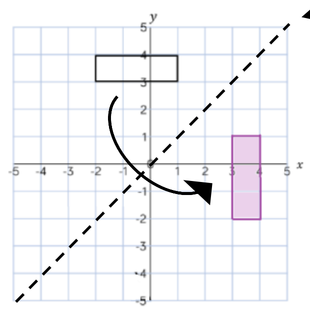




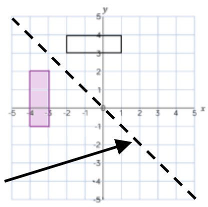
Drawing perpendicular lines
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

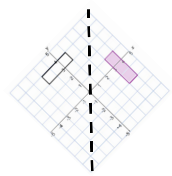
Reflect Diagonally (2)

This is the line **y = x** (every y coordinate is the same as the x coordinate along this line)



This is the line **y = - x**
The x and y coordinate have the same value but opposite sign





Turn your image
If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)

YEAR 9 - REASONING WITH GEOMETRY...

Rotation & Translation

What do I need to be able to do?

By the end of this unit you should be able to:

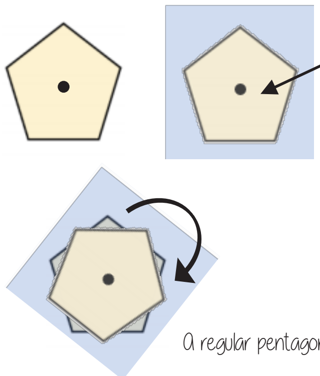
- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement.
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.
Vertex: a point two edges meet.
Horizontal: from side to side
Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



1 Trace your shape (mark the centre point)

2 Rotate your tracing paper on top of the original through 360°

3 Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

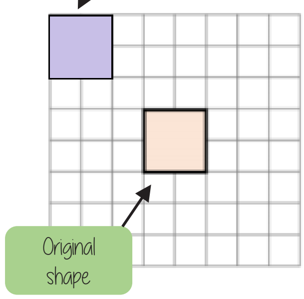
Translation and vector notation

Vector Notation → $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

How far left or right to move
Negative value (left)
Positive value (right)

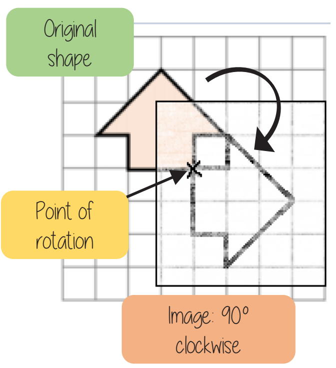
How far up or down to move
Negative value (down)
Positive value (up)

Translation $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$



Every vertex has been translated by the same amount

Rotate from a point (in a shape)

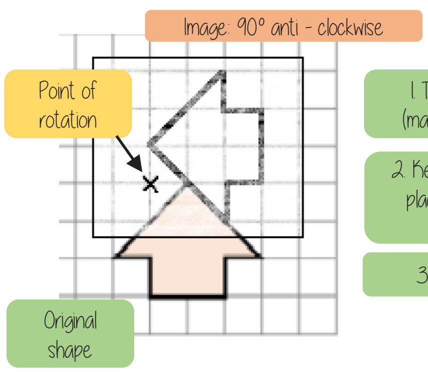


1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape

Rotate from a point (outside a shape)

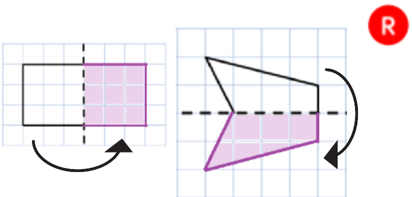


1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

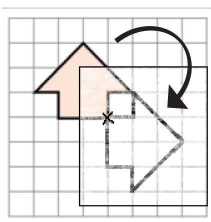
3 Draw the new shape

Compare rotations and reflections



Reflections are a mirror image of the original shape.

Information needed to perform a reflection:
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:
- Point of rotation
- Direction of rotation
- Degrees of rotation

YEAR 9 - REASONING WITH GEOMETRY...

Enlargement & Similarity

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes

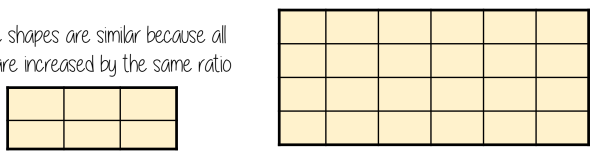
Keywords

Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.
Scale Factor: the multiple describing how much a shape has been enlarged
Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)
Corresponding: objects (or sides) that appear in the same place in two similar situations.
Image: the picture or visual representation of the shape

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

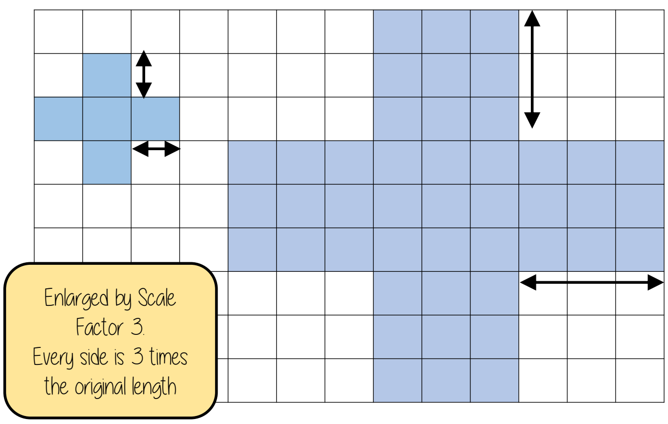
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

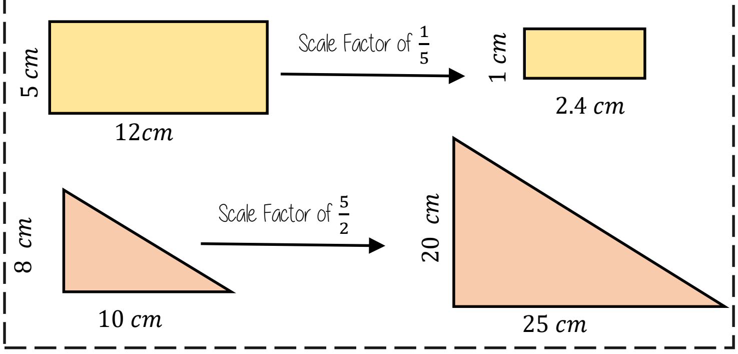
Enlarge by a positive scale factor

With a scale factor larger than 1 it makes the shape **bigger**



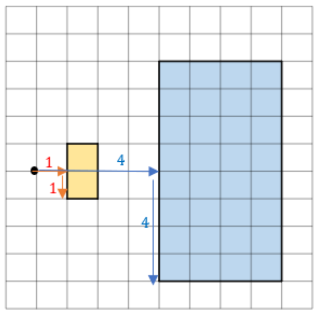
Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape **smaller**



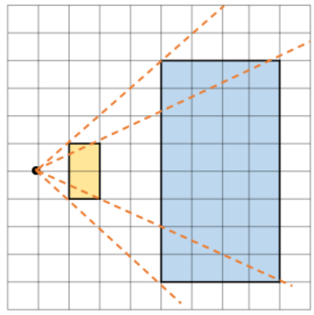
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

Rays method

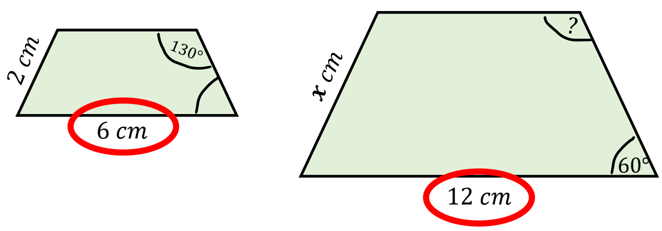


Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



<u>Corresponding sides identify the scale factor</u> $\frac{12}{6} = 2$ Scale Factor = **2**

<u>Calculate the missing side</u> Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

<u>Calculate the missing angle</u> Corresponding angles remain the same

$130°$

YEAR 9 – CONSTRUCTING IN 2D/3D…
3D Shapes

What do I need to be able to do?

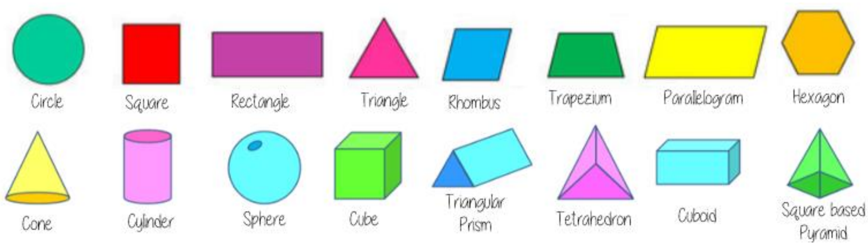
By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

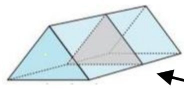
2D: two dimensions to the shape e.g. length and width
3D: three dimensions to the shape e.g. length, width and height
Vertex: a point where two or more line segments meet
Edge a line on the boundary joining two vertex
Face: a flat surface on a solid object
Cross-section: a view inside a solid shape made by cutting through it
Plan: a drawing of something when drawn from above (sometimes birds eye view)
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

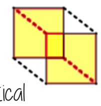
Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

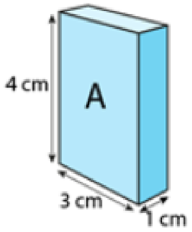


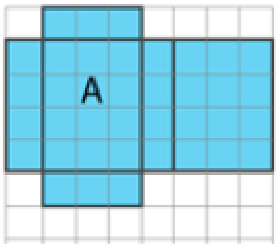


The cross section will also be identical to the end faces.

A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

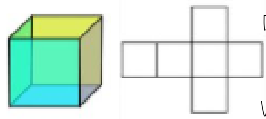




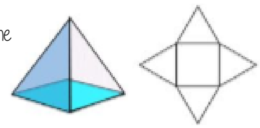
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

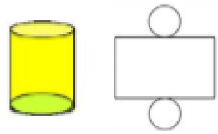
Sketch and recognise nets



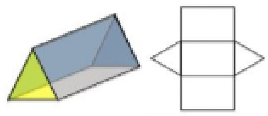
Do they have the same number of faces?



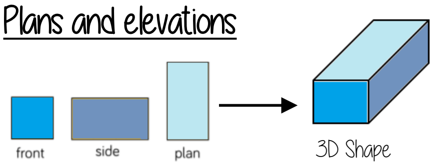
Where do the edges join?



Are the shapes of the faces correct?



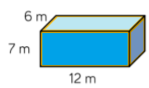
Plans and elevations

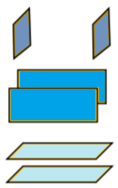


The direction you are considering the shape from determines the front and side views

Surface area

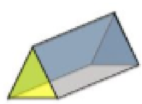
Sketching nets first helps you visualise all the sides that will form the overall surface area





Sides	6 x 7	Sum of all sides is surface area
	6 x 7	
Front and back	12 x 7	
	12 x 7	
Top and Bottom	12 x 6	
	12 x 6	

For cubes and cuboids you can also find one of each face and double it



For other shapes = not all the sides are the same, so calculate the individually

Volumes

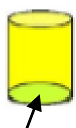
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes

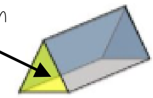
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



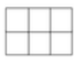
Cross section

Prisms and cylinders = area cross section x height

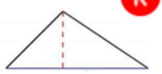
Height can also be described as depth

Area of 2D shapes

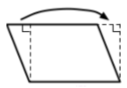
Rectangle
Base x Height



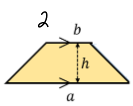
Triangle
½ x Base x Perpendicular height



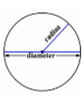
Parallelogram/ Rhombus
Base x Perpendicular height



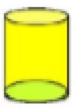
Area of a trapezium
$\frac{(a+b) \times h}{2}$

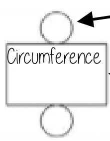


Area of a circle
π x radius^2



Surface area - cylinders





The area of the circle
π x radius^2

The width of this face is the same as the circumference
π x diameter x height

2 x π x radius^2 + π x diameter x height

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 9 - CONSTRUCTING IN 2D/3D...
Constructions & congruency

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

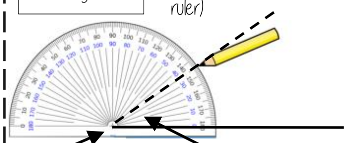
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) – a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil
And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

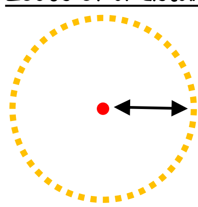
A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life
$\times 10$ 1cm : 30cm $\times 10$
10cm : 300cm

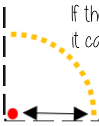
Locus of a distance from a point



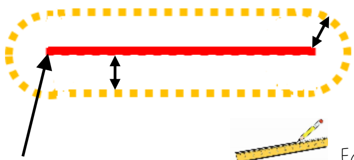
All points are equidistant (the same distance) from the fixed point in the middle.

Equipment needed
The radius is the distance from the fixed point

If the point is in the corner it can only make a quarter circle



Locus of a distance from a straight line

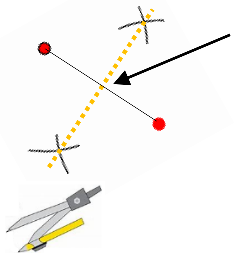


All points are equidistant (the same distance) from line

The ends of the line are fixed points

Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

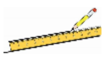
Locus equidistant from two points



Also a perpendicular bisector

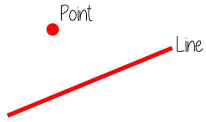
Because if the points are joined, this new line intersects it at a 90°

Keep the compass the same size and draw two arcs from each point

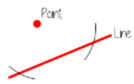


Join the intersections with a ruler.
All points on this line are equidistant from both points

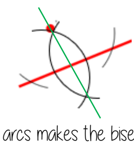
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



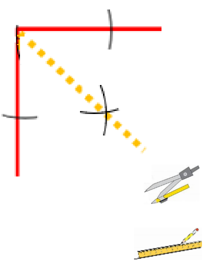
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

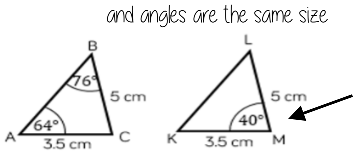
Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

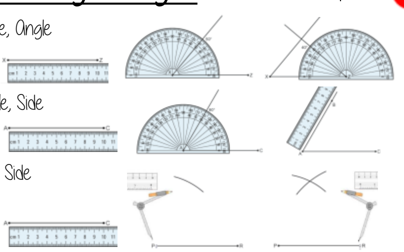
Constructing Triangles

Link to steps R

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

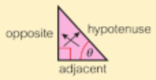
Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 9 – Trigonometry

Using sine to find sides and angles

Key point
The side opposite the chosen angle (angle θ in this diagram) is called the **opposite** side. The side next to θ is called the **adjacent** side.

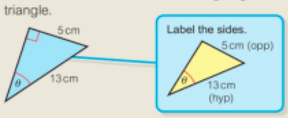


Key point
The ratio of the opposite side to the hypotenuse is called the **sine** of the angle.
The sine of angle θ is written as **sin θ**.
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

Key point
You can use **inverse** trigonometric functions to work out unknown angles.
$\sin\theta = x$, so $\theta = \sin^{-1} x$

$S\frac{O}{H}\,C\frac{A}{H}\,T\frac{O}{A}$

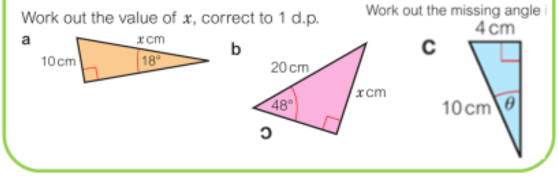
Worked example
Use the sine ratio to find the missing angle in this right-angled triangle.



Using the sine ratio
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
$\sin\theta = \frac{5}{13}$
$\theta = 22.6°$

You need to find $\sin^{-1}\frac{5}{13}$
Use these buttons on your calculator:
SHIFT sin 5 ÷ 1 3 =

Questions
Work out the value of x, correct to 1 d.p.



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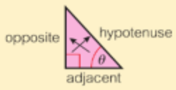
Answers: **a** 32.4 **b** 14.9 **c** 23.6°

Using cosine to find sides and angles

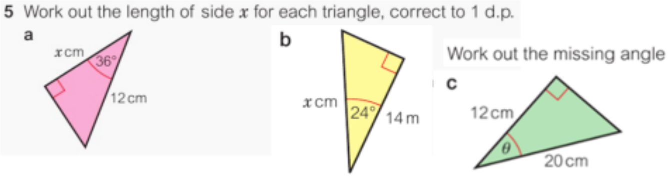
Key point
The side opposite the chosen angle (angle θ in this diagram) is called the **opposite** side. The side next to θ is called the **adjacent** side.



Key point
The ratio of the adjacent side to the hypotenuse is called the **cosine** of the angle.
The cosine of angle θ is written as **cos θ**.
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

Key point
You can use **inverse** trigonometric functions to work out unknown angles.
$\cos\theta = x$, so $\theta = \cos^{-1} x$
For examples look at 9.1 'using tan' and 9.2 'using sine'

Questions:
5 Work out the length of side x for each triangle, correct to 1 d.p.



Answers:
a 9.7
b 15.3
c 53.1°

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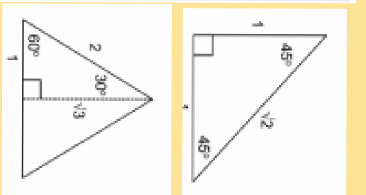
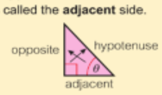
Exact Trig values

Angle (θ)	sin(θ)	cos(θ)	tan(θ)
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined



Using tangent to find sides and angles

Key point
The side opposite the chosen angle (angle θ in this diagram) is called the **opposite** side. The side next to θ is called the **adjacent** side.

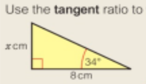


Key point
The ratio of the opposite side to the adjacent side is called the **tangent** of the angle.
The tangent of angle θ is written as **tan θ**.
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Key point
You can use **inverse** trigonometric functions to work out unknown angles.
$\tan\theta = x$, so $\theta = \tan^{-1} x$
For a worked example to find a missing angle in a right-angle triangle, look at 9.2 'using sine'

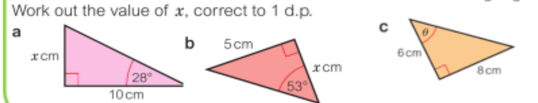
Hint for Qb:
$\text{Tan}53 = \frac{5}{x}$
Rearranges to
$x = \frac{5}{tan53}$

Worked example
Use the **tangent** ratio to work out the value of x, correct to 1 d.p.



$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ — Write the tangent ratio.
opposite = x
adjacent = 8 — Identify the opposite and adjacent sides.
$\theta = 34°$
$\tan 34° = \frac{x}{8}$ — Substitute the sides and angle into the equation.
$8 \times \tan 34° = x$ — Rearrange to make x the subject. Use your calculator to work out $8 \times \tan 34°$.
$x = 5.4$ cm (to 1 d.p.)

Questions
Work out the value of x, correct to 1 d.p.



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Answers:
a 5.3
b 3.8
c 53.1°

YEAR 9 - REPRESENTATIONS...

Probability

What do I need to be able to do?

By the end of this unit you should be able to:

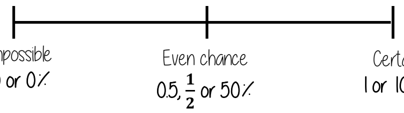
- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

Keywords

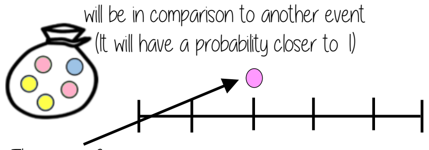
Probability: the chance that something will happen
Relative Frequency: how often something happens divided by the outcomes
Independent: an event that is not effected by any other events.
Chance: the likelihood of a particular outcome.
Event: the outcome of a probability – a set of possible outcomes.
Biased: a built in error that makes all values wrong by a certain amount.

The probability scale

R



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Single event probability

R

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(white chocolate) = 1 – 0.15 – 0.35
= 0.5

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times
$0.3 \times 100 = 30$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

On experiment is carried out 400 times.
Show that dark chocolate is expected to be selected 60 times

$0.15 \times 400 = 60$

Independent events

The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

Probability of event 1 × Probability of event 2

Find the probability of getting a 5 and a red

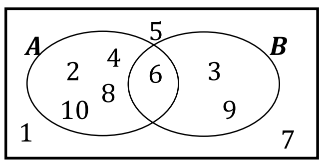
$P(5) = \frac{1}{6}$ $\quad P(R) = \frac{1}{4}$

$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables

R



	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T