

Algebra - Foundation

Notation

$ab = a \times b$
 $a^2 = a \times a$
 $(2a)^3 = 2a \times 2a \times 2a$
 $(a + b)^2 = (a + b)(a + b)$

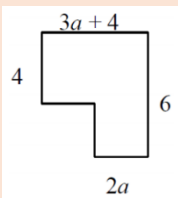
Definitions

Expression – no equal signs e.g. $2x + 3$, $2y$, $(3x - 2)^2$
Equations – equal signs, can be solved, e.g. $y + 4 = 10$
Identities – identical/equivalent to e.g. $2(y + 4) \equiv 2y + 8$
Formulae – equal signs, more than one unknown e.g. $A = \frac{1}{2}bh$

Simplifying expressions by collecting like terms

Always circle the sign IN FRONT of the term to avoid errors.

$$(3x) - (7b) - (x) + (9b) \equiv 2x + 2b$$



Typical Exam Q: Create an expression for the perimeter of the shape by adding and collecting like terms.
If the perimeter is given as 20cm, for example, you can create an equation:

$$4 + 3a + 4 + 6 + 2a = 20$$

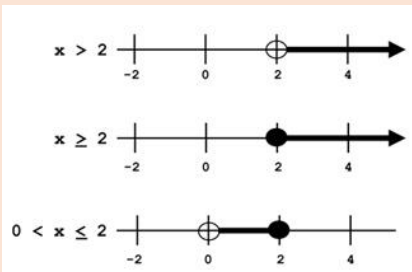
$$5a + 14 = 20$$

Simplifying expressions multiplication and division

$$2ma^2 \times 7ma = 14m^2a^3$$

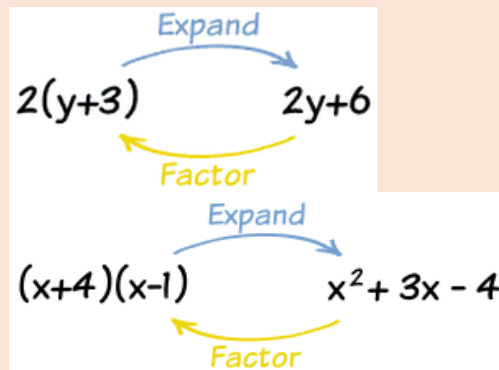
$$\frac{18b^6}{3ab^2} = \frac{6b^4}{a}$$

Inequalities

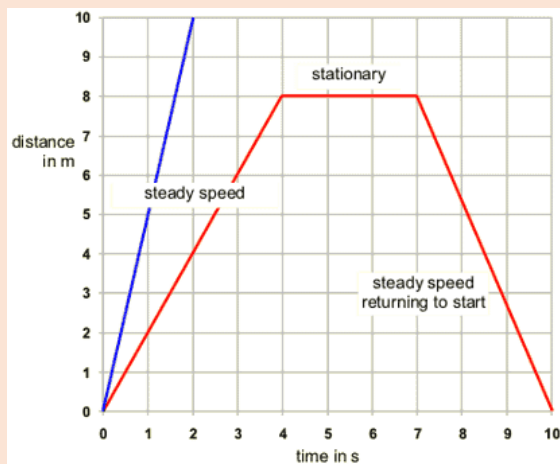


Open circle: $</>$
Closed circle: \leq/\geq

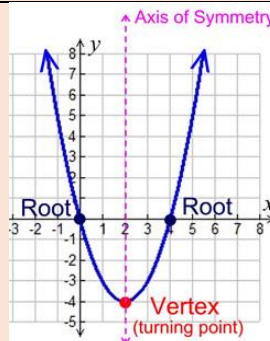
Factorising and expanding



Distance / Time Graphs

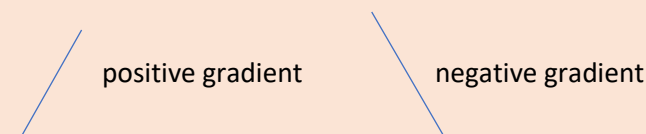


Turning point and roots of a quadratic equation



Straight line graphs

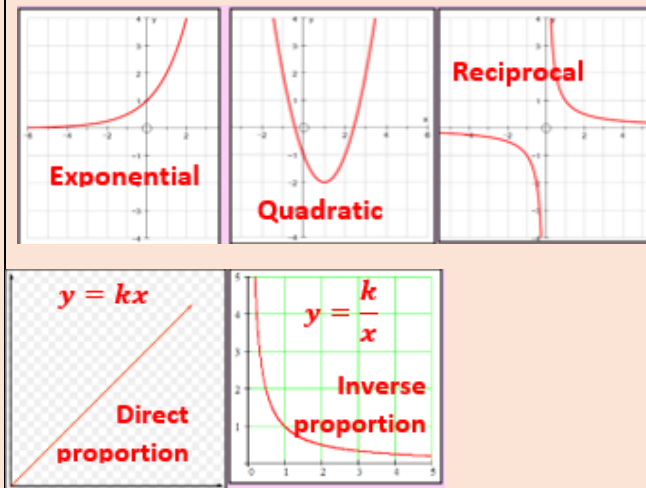
$y = mx + c$
 $m = \text{gradient}$
 $c = y - \text{intercept}$



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

Parallel lines – have equal gradients

Graphs that need to be recognised



Finding the nth term of a linear sequence

- 5, 7, 9, 11, 13, ...
- Find the common difference: 2
 - This is the coefficient of n: $2n$
 - Find the difference between the coefficient of n and the first term $5 - 2 = 3$
 - Add this to the amount of n
 $2n + 3$

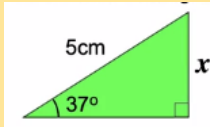
Trigonometry

$$S \frac{O}{H} C \frac{A}{H} T \frac{O}{A}$$

Example – finding a side:

$$\sin 37 = \frac{x}{5}$$

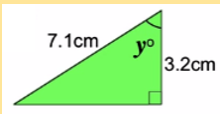
$$x = 5 \times \sin 37^\circ$$



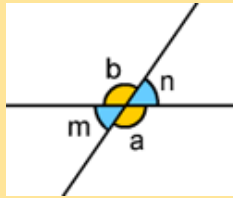
Example – finding a side:

$$\tan y = \frac{3.2}{7.1}$$

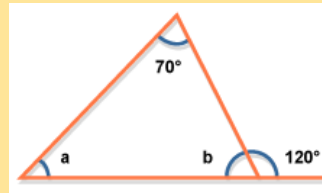
$$y = \tan^{-1}\left(\frac{3.2}{7.1}\right)$$



Angle Facts



Vertically opposite angles are equal: $a=b$ and $m=n$

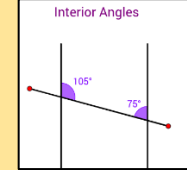
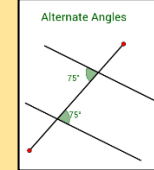
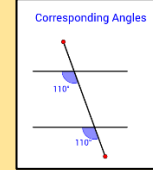


Angles in a triangle sum to 180° .

Angles on a straight line sum to 180° .

E.G: $b=60^\circ$ so $a = 50^\circ$

Angles in parallel lines



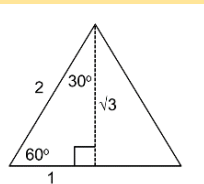
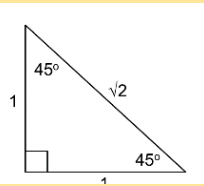
Corresponding angles are equal

Alternate angles are equal

Co-interior angles add to 180.

Exact Trig values

| Angle (θ) | $\sin(\theta)$ | $\cos(\theta)$ | $\tan(\theta)$ |
|--------------------|----------------------|----------------------|----------------------|
| 0° | 0 | 1 | 0 |
| 30° | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{3}}$ |
| 45° | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 1 |
| 60° | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| 90° | 1 | 0 | undefined |



Simple vector notation

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

a : movement along the x-axis (left or right)

b : movement along the y-axis (up or down)

$-a$: movement left

$-b$: movement down

Operations with vectors

$$\begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} 7 \\ -3 \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$$

If $b = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$, then $3b = \begin{pmatrix} 12 \\ -6 \end{pmatrix}$

Volume & surface area

Volume = area of cross section x length

Surface area = area of all the faces of a 3D shape

Learn the cylinder

$$V = \pi r^2 h$$

$$SA = 2\pi r^2 + \pi dl$$

Types of triangles

- Right angled
- Isosceles
- Equilateral
- Scalene

Types of quadrilaterals

- Square
- Rectangle
- Parallelogram
- Rhombus
- Trapezium
- Kite

Area of key shapes

Triangle: $A = \frac{b \times h}{2}$ (h = perpendicular height)

Parallelogram: $A = b \times h$ (h = perpendicular height)

Trapezium: $A = \left(\frac{a+b}{2}\right) \times h$ (add together the parallel sides, divide the total by 2, and then multiply by the perpendicular height between the parallel sides)

Angles in regular polygons



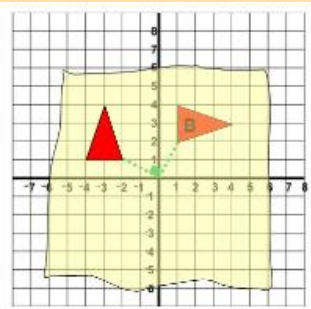
n = number of sides

Interior angle + exterior angle = 180°

$$\text{Exterior angle} = \frac{360}{n}$$

$$n = \frac{360}{\text{Exterior angle}}$$

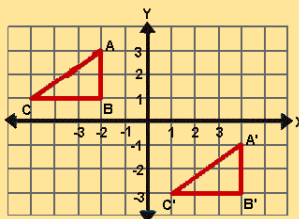
Transformations – rotation



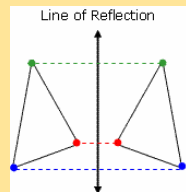
Always use tracing paper.
Describe:

1. It's a rotation
2. Size of rotation in degrees
3. Orientations: clockwise or anticlockwise
4. Centre of rotation given as a coordinate (x,y)

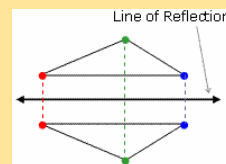
Transformations – translations and reflections



Translate triangle ABC to A'B'C' with the vector $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

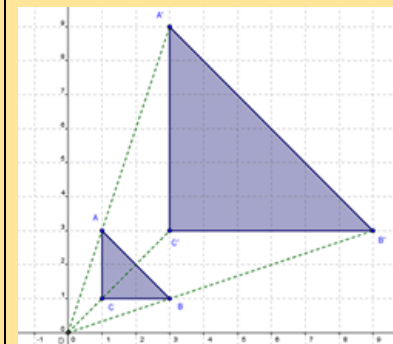


Reflection in the line $x=a$



Reflection in the line $y=a$

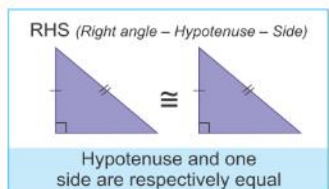
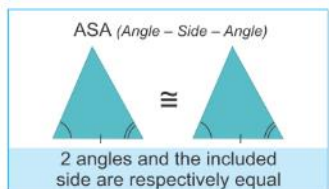
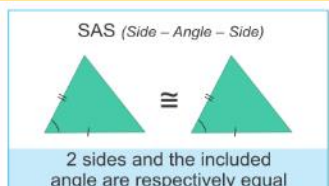
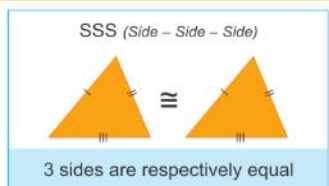
Transformations - enlargement



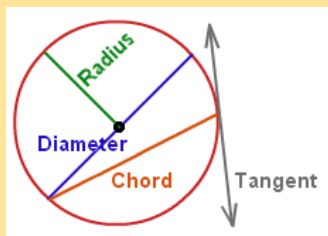
Describe:

1. It's an enlargement
2. The scale factor (if the image is smaller than the object the scale factor is fractional e.g. $\frac{1}{2}$)
3. The centre of enlargement given as a coordinate

Congruent triangles

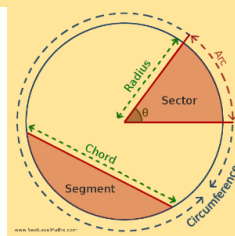


Circles



$$\text{Area} = \pi r^2$$

$$\text{Circumference} = \pi d$$



$$\text{Sector Area} = \frac{\theta}{360} \pi r^2$$

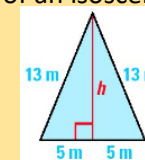
$$\text{Arc length} = \frac{\theta}{360} \pi d$$

Pythagoras' Theorem

$$a^2 + b^2 = c^2$$

Only applies to right angled triangles.

Can be used to find the height of an isosceles triangle

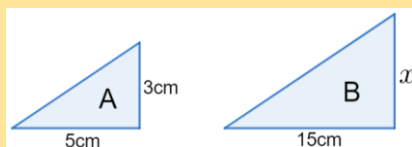


Can be used to find the length distance between two coordinates

Similar shapes

Same shape, different sizes

The ratio of the lengths of corresponding sides are equal



$$\text{Length scale factor} = 15 \div 5 = 3$$

$$x = 3 \text{ cm} \times 3$$

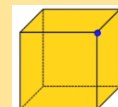
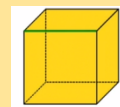
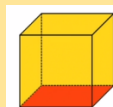
3D notation

Cube:

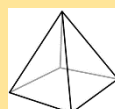
Faces: 6

Edges: 12

Vertices: 8



Square based pyramid:



$$F = 5, E = 8, V = 5$$

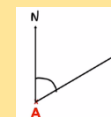
Bearings

Measure from the North

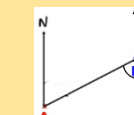
Measured in a clockwise direction

Written using 3 digits


Bearing of B **from** A (start at A)

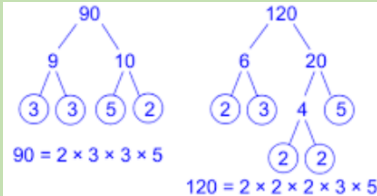
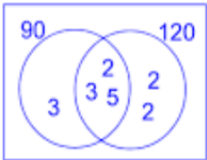


Bearing of A **from** B (start at B)



Number Ratio and Proportion - Foundation

| | | |
|---|---|--|
| <p>Estimate Round each value to one significant figure</p> <p>Standard form $a \times 10^n$, where $1 \leq a < 10$</p> | <p>Simplifying Ratio Divide both sides by the highest common factor</p>  | <p>Percentages</p> <p>Finding percentages of an amount</p> <p>1% $\div 100$ 5% $\div 20$ 20% $\div 5$ 25% $\div 4$ 50% $\div 2$</p> <p>Multipliers: To find the multiplier for a percentage, divide by 100</p> <p>Use multipliers on a calculator paper e.g. 35% of 370 = 0.35×370</p> <p>Increasing and decreasing a given amount Calculator: $Original\ Amount \times multiplier = new\ amount$</p> <p>Non-calculator: find the increase or decrease and add to the original amount</p> <p>Finding percentage increase or decrease (profit/loss) $\frac{value\ of\ increase/decrease}{Original} \times 100$</p> <p>Writing an amount as a percentage of the original $\frac{Amount}{Original} \times 100$</p> <p>Reverse Percentage – finding the original amount</p> $Original\ Amount = \frac{New\ Amount}{multiplier}$ |
| <p>Reciprocal Reciprocal of 7 is $\frac{1}{7}$, reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ etc</p> <p>Sequences Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21 Geometric Sequence: each term is multiplied but he same constant to get the next number. E.g. 3, 12, 48, 191, (x by 4 each time)</p> | <p>Simplifying Ratio 1:n Divide both sides by the highest factor of the left hand side</p> <p>2m: 180cm 200cm: 180cm 2:1.8 1: 0.9</p> | |
| <p>Squares and Cubes Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225 etc</p> <p>Cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, etc</p> | <p>Fractions</p> <p>Add and Subtract – ensure the fractions have the same denominator before adding numerators</p> $\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$ | |
| <p>Sharing in a given Ratio</p> <p>A Add the ratio parts D Divide the amount by the total parts A and M Multiply the ratio by the value of one part</p> <p>e.g. share £420 in the ratio 2:5</p> $2 + 5 = 7$ $420 \div 7 = £60$ <p>2: 5</p> <p>(x60) (x60) £120 : £300</p> | <p>Multiply – multiply numerators and denominators</p> $\frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$ <p>Divide – take reciprocal of the second fraction and then multiply the new numerators and denominators</p> $\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \times \frac{3}{1} = \frac{12}{5} = 2\frac{2}{5}$ | |

| | | |
|---|---|--|
| <p>Growth & Decay / Compound interest</p> <p>$original\ amount \times multiplier^{time}$</p> <p>Where the multiplier is the percentage, increase or decrease from 100%, converted to a decimal. e.g. 30% decrease is 70% = 0.7 30% increase is 130% = 1.3</p> | <p>Dividing by decimals:</p> <ol style="list-style-type: none"> 1. Write the calculation as a fraction 2. Form an equivalent fraction to makes integers (multiply by powers of 10) 3. Use short division (bus stop) to calculate <p>e.g. $460 \div 0.4 = \frac{460}{0.4} = \frac{4600}{4} = 1150$</p> | <p>Conversions</p> <p>10 millimetres = 1 centimetre 15 minutes = 0.25 hours 100 centimetres = 1 metre 30 minutes = 0.5 hours 1000 metres = 1 kilometre 45 minutes = 0.75 hours 1000cm³ = 1 litre 1000g = 1 kilogram 1000ml = 1 litre 1000kg = 1 tonne</p> |
| <p>Compound Units (rearrange as necessary)</p> $Speed = \frac{Distance}{Time}$ $Area = \frac{Force}{Pressure}$ $Density = \frac{Mass}{Volume}$ | <p>Error Intervals least possible value $\leq x <$ greatest possible value</p> <p>e.g. A fence is 30 m long to the nearest 10 m. $25\ m \leq l < 35\ m$</p> <p>Truncation Truncation is a method of approximating a decimal number by dropping all decimal places past a certain point without rounding.</p> <p>e.g. Truncate 3.14159265 to 4 decimal places. $= 3.1415$</p> | <p>Negative numbers <u>Adding and subtracting: (vertical number lines help)</u></p> <p>$-3 - 5 = -8$ $-3 + 5 = 2$ $-3 - -5 = -3 + 5 = 2$ $-3 - +5 = -3 - 5 = -8$ $-3 + -5 = -3 - 5 = -8$</p> <p><u>Multiplying and dividing:</u> Different signs – answer will be negative $+x - = -$, $-x + = -$ Same signs – answer will be positive $-x - = +$</p> |
| <p>Ordering fractions Calc: use division to write each fraction as a decimal Non-calc: write fractions with common denominators</p> | <p>Order of operations Bracket Indices Division and Multiplication Addition and Subtraction</p> | <p>Rounding to significant figures Start from the first non-zero number and round as normal, but ensure the place value is correct e.g. 345,635 to 2SF = 350,000 0.0060821 to 3SF = 0.0608</p> |
| <p>Index Laws</p> $a^n \times a^m = a^{n+m}$ $a^n \div a^m = a^{n-m}$ $(a^n)^m = a^{nm}$ $a^0 = 1$ $a^{-n} = \frac{1}{a^n}$ $\frac{n}{a^m} = \frac{1}{a^{\frac{m}{n}}}$ | <p>Prime Factorisation</p>  | <p>HCF and LCM of 90 and 120 (Factor Tree & Venn Diagram) HCF is the product of common factors LCM is the product of common factors and remaining factors.</p>  <p>HCF: $2 \times 3 \times 5$ LCM: $2^3 \times 3^2 \times 5$</p> |

Probability and Statistics - Foundation

Averages

Mode: most common piece of data

Mean: Sum of the data \div total frequency

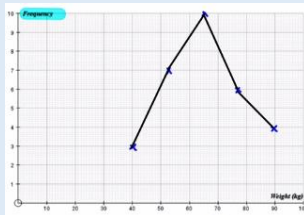
Median: order the data and find the middle value

Range: Highest value – lowest value

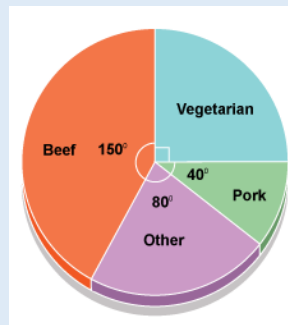
Frequency Polygons

- Plot frequency at the mid-point
- Join with straight lines

| Weight w (kg) | Frequency |
|-------------------|-----------|
| $30 \leq w < 50$ | 3 |
| $50 \leq w < 55$ | 7 |
| $55 \leq w < 75$ | 10 |
| $75 \leq w < 80$ | 6 |
| $80 \leq w < 100$ | 4 |



Reading and Drawing Pie Charts



Find the fraction of the total

1000 people were surveyed

$$\text{Beef: } \frac{150}{360} \times 1000$$

$$\text{Vegetarian: } \frac{90}{360} \times 1000$$

| Hair colour | People |
|-------------|--------|
| Blonde | 8 |
| Brown | 12 |
| Red | 3 |
| Grey | 2 |
| Black | 6 |

Find the fraction of the full circle.

Size of Blonde sector:
 $\frac{8}{31} \times 360^\circ$

Averages from a frequency table

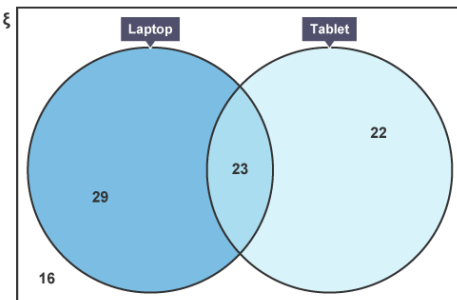
Mean: $\frac{\sum fw}{\sum f}$; where, w is the midpoint of the group.

Median group: find which group the $\frac{n+1}{2}$ th, value lies. Where, n is the total frequency.

E.G. in this table 51.5th value which lies in group $8 < w \leq 12$ (using the cumulative frequency)

| Weight of box (w kg) | Frequency |
|-------------------------|-----------|
| $0 < w \leq 4$ | 11 |
| $4 < w \leq 8$ | 16 |
| $8 < w \leq 12$ | 29 |
| $12 < w \leq 16$ | 26 |
| $16 < w \leq 20$ | 20 |

Venn Diagrams



Information given:
 90 pupils were surveyed
 52 said they owned a laptop.
 45 said they owned a tablet.
 23 said they owned both.

Expected outcomes

Expected outcome = probability \times number of trials

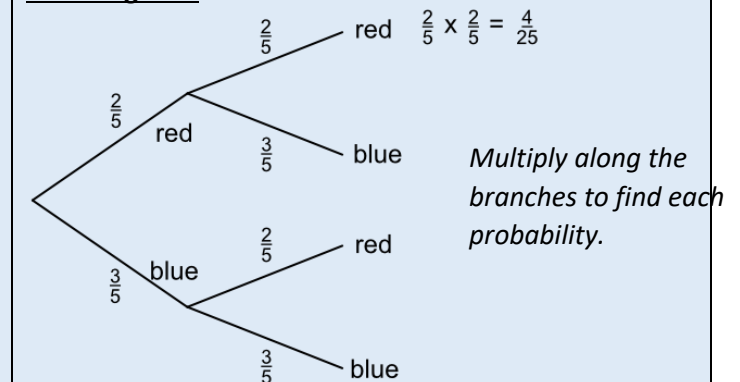
E.g. A biased spinner is spun 800 times. The probabilities it lands on each colour is below. The probability of it landing on red is the same as the probability of it landing on green. How many times would you expect yellow to come up.

| Result | Red | Green | Brown | Yellow |
|-------------|-----|-------|-------|--------|
| Probability | | 0.48 | 0.2 | |

$$P(Y) = (1 - 0.48 - 0.2) \div 2 = 0.32 \div 2 = 0.16$$

$$\text{Expected yellow} = 0.16 \times 800 = 128$$

Tree diagrams



Multiply along the branches to find each probability.

- Probability that a red counter is picked both times $P(RR) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$
- Probability that the counters are different colours = $P(RB) + P(BR) = \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} = \frac{12}{25}$

Probability Definitions

Total probability: adds to 1

Relative frequency: *frequency \div total trials*

Independent events: one event doesn't impact the other