

1) All numbers can be written as a product of primes. This is called **prime factor decomposition**.

Use prime factor decomposition to find the HCF or LCM of a set of numbers.

**Index notation** collects factors together and writes them as a power,

e.g.  $2 \times 2 \times 2 \times 2 \times 2 = 2^5$ . index notation saves space.

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2) **Index Laws** You can simplify expressions containing powers to make calculations easier.

$$x^a \times x^b = x^{a+b}$$

$$x^a \div x^b = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

These three rules are called the Multiplication, Division, and Power Laws.

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3) **Powers of 10**

$$10^0 = 1 \qquad 10^{-1} = 0.1$$

$$10^1 = 10 \qquad 10^{-2} = 0.01$$

$$10^2 = 100 \qquad 10^{-3} = 0.001$$

$$10^3 = 1000 \qquad 10^{-4} = 0.0001$$

$$10^4 = 10000 \qquad 10^{-5} = 0.00001$$

$$10^5 = 100000 \qquad 10^0 = 100000$$

Negative powers do not mean negative numbers!

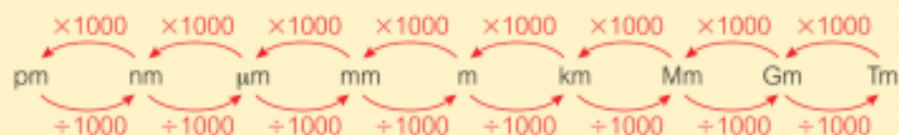
Any number with an index of 0 is equal to 1.

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4) **Key point**



← To convert bigger units to smaller units, multiply



To convert smaller units to bigger units, divide →

5) ○ An **expression** contains letter and/or number terms, but no equals sign, e.g.  $2ab$ ,  $7x + 3xy$ ,  $5st - 9$ .

○ An **equation** has an equals sign, letter terms, and numbers. You can solve it to find the value of the letter, e.g.  $2x - 4 = 9x + 1$ .

○ An **identity** is true for all values of the letters, e.g.  $x(x+y) \equiv x^2 + xy$

○ A **formula** has an equals sign and letters to represent different quantities, e.g.  $v^2 = u^2 + 2as$ .

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6) ○ **Show that** means "show your working".

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○ To **factorise** an expression completely, take out the highest common factor of its terms.

○ **Substitute** means replace letters with given numbers.

○ A **linear expression** is one where the highest power is 1.

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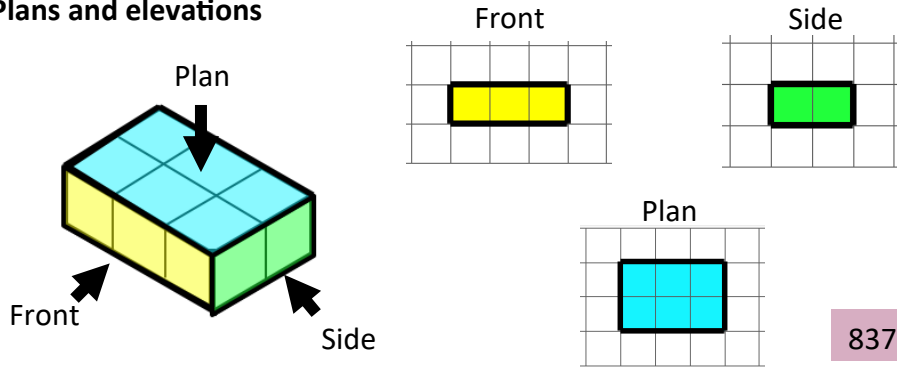
7) **Evaluate** means "work out the value".

You can **round** to a given number of **decimal places (d.p.)** This means there should be a certain number of digits after the decimal point.

You can round to a given number of **significant figures (s.f.)**. The first significant figure is the first non-zero digit in the number, counting from the left.

MathsWatch clips 32 and 90

**Plans and elevations**



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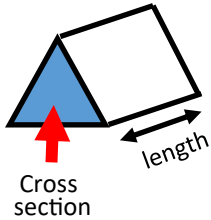
**Direct proportion**

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For two quantities to be in direct proportion their graph must:

- start at the origin (0,0)
- be a straight line
- Show that when one quantity doubles, so does the other.

**3D shapes**



A **prism** is a solid with the **same cross section** throughout its length. The cross section can be any flat shape.

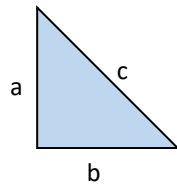
**Surface area** of a prism = the total area of each individual face.

**Volume** of a prism = the area of the cross section x length

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**Pythagoras Theorem**

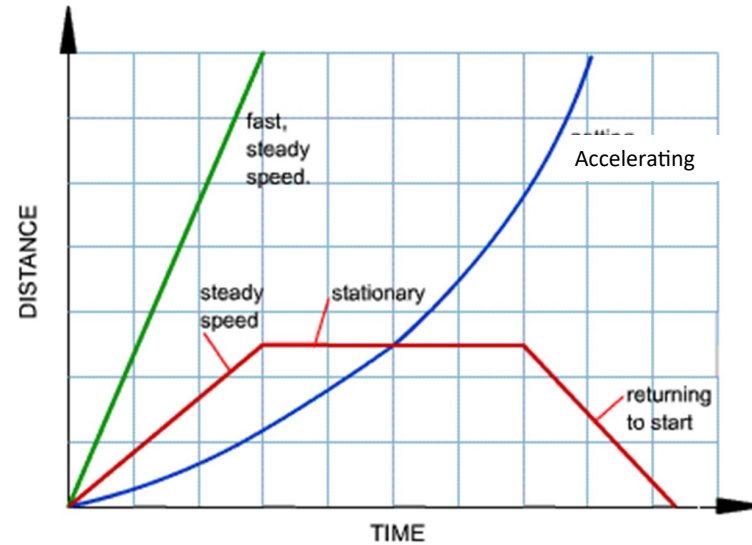
Pythagoras theorem can be used to find a missing length of a **right-angled** triangle.



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**Distance-time graphs**

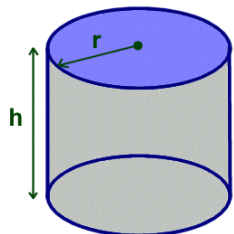


**Circles and cylinders**

**Circumference** is the perimeter of a circle.

An **arc** is part of the circumference.

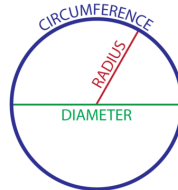
A **sector** is the part of a circle enclosed by 2 radii.



**Formulae:**

$Circumference = \pi d$

$Area\ of\ a\ circle = \pi r^2$



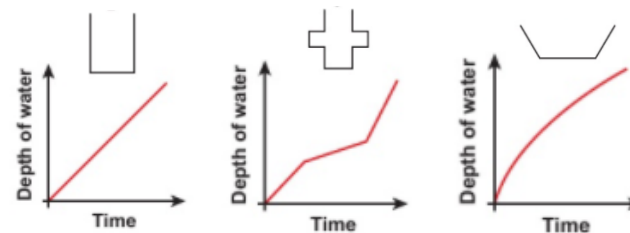
$Surface\ area\ of\ a\ cylinder = 2\pi r^2 + 2\pi rh$

$Volume\ of\ a\ cylinder = \pi r^2 h$

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**Rates of change**

These rates of change graphs show the time taken against the depth for each of the following vases when they are filled with water.



A **rate of change** graph shows how a quantity changes over time.

### 1) Transformations

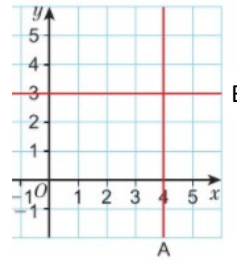
Transformation	Information needed
Reflection	1. Equation of the mirror line
Rotation	1. Angle of rotation 2. Direction of rotation 3. Centre of rotation
Enlargement	1. Scale factor 2. Centre of enlargement
Translation	1. Translation vector

You may be asked to perform a transformation, in which case you will need:

- Pencil
- Ruler
- Tracing paper

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You may be asked to describe a transformation. In this case give the information in the table.

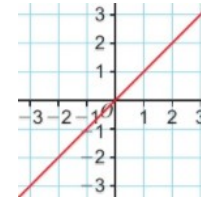


With **vertical** straight line graphs, their x co-ordinate is always the same so they are parallel to the **y axis**.

With **horizontal** straight line graphs, their y co-ordinate is always the same so they are parallel to the **x axis**.

For example: **A: x = 4**

**B: y = 3**

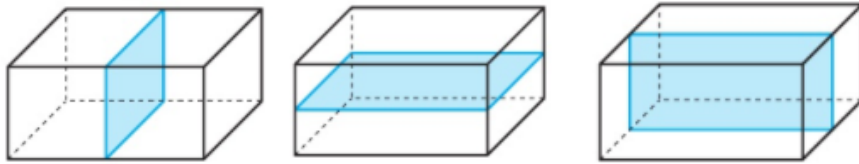


**y = x**

A diagonal line through the origin where the co-ordinates share the same x and y value.

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### 2) 3D shapes



A 2D shape can have lines of symmetry, a 3D shape can have planes of symmetry. On either side of the plane of symmetry the solid is identical.

### 5) Percentage of amounts

To convert from a percentage to a decimal we divide by 100. This is called a multiplier because it can be used to find a percentage of an amount.

For example: Find 35% of £80

$$35 \div 100 = 0.35$$

Multiplier

$$0.35 \times 80 = \text{£}28$$

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### 3) Recurring decimals

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$$0.\dot{6} \rightarrow 0.66666666$$

$$0.5\dot{3} \rightarrow 0.53333333$$

$$0.\dot{7}\dot{2} \rightarrow 0.72727272$$

$$0.\dot{4}7\dot{9} \rightarrow 0.47947947$$

$$0.3\dot{2}\dot{8} \rightarrow 0.32828282$$

In **recurring decimals** the numbers repeat in a pattern forever, the pattern is determined by the position of the dots over the numbers.

#### Useful recurring decimals

$$\frac{1}{3} = 0.3333 \dots \quad \frac{1}{9} = 0.1111 \dots \quad \frac{1}{6} = 0.1666 \dots$$

### 6) Percentage change

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$$\text{Percentage change} = \frac{\text{actual change}}{\text{original amount}} \times 100$$

### 7) Compound interest

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In **compound interest**, the interest earned in each year is added to money in the account and earns interest the next year. Most interest rates are compound interest rates.

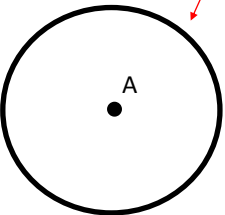
You can calculate an amount after  $n$  years' compound interest using the formula:

$$\text{Amount} = \text{initial amount} \times \left( \frac{100 + \text{interest rate}}{100} \right)^n$$

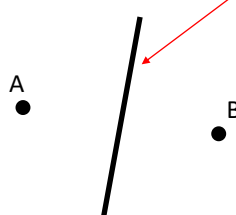
1) Construct	Draw accurately with a ruler and compasses.
Perpendicular	Meeting at a right angle (90°).
Perpendicular bisector	Cuts a line in half at right angles.
Angle bisector	Cuts an angle exactly in half.
Locus	A set of all points that obey a certain rule. Often the locus is a path.
Equidistant	The same distance.

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2) The locus of points equidistant from point A is a circle.

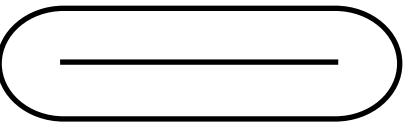


The locus of points equidistant from two points is the perpendicular bisector.

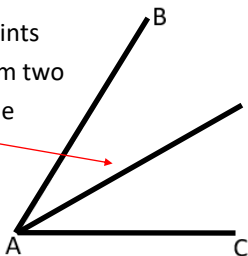


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The locus of points equidistant from a line is a pair of parallel lines and two semicircles.



The locus of points equidistant from two lines is the angle bisector.



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3) Probability Scale	All probabilities have a value between 0 and 1.
Outcome	An outcome is an end result, e.g. heads when flipping a coin.
Fair	All outcomes are equally likely. Each player has an equal chance of winning.
Random	Each item has the same chance of being picked.
Mutually exclusive	Events which cannot happen at the same time.

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4) Probability of an event happening =  $\frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$

Theoretical probability is calculated without doing an experiment.

Relative frequency of a value =  $\frac{\text{Frequency of value}}{\text{Total frequency}}$

Probability can be used to model what happens in the future.

Experimental probability of an outcome =  $\frac{\text{Number of successful outcomes}}{\text{Total number of outcomes}}$

More data = more confidence

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5) Sample Space Diagram

- shows all the possible outcomes of two events
- Is a table

Two fair spinners are numbered 1,1,1,2,3 and 1,2,2,3,3. The sum of the two numbers is calculated.

a) Find the probability that the sum is exactly 6  
b) Find the probability that the sum is less than 6

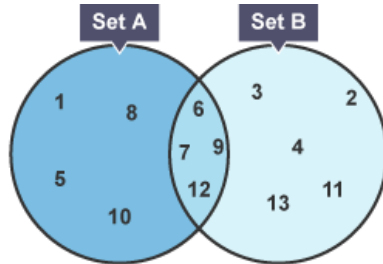
+	1	1	1	2	3
1	2	2	2	3	4
2	3	3	3	4	5
2	3	3	3	4	5
3	4	4	4	5	6
3	4	4	4	5	6

$P(6) = \frac{2}{25}$

$P(<6) = \frac{23}{25}$

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6) Venn Diagrams



Set A = {1, 5, 6, 7, 8, 9, 10, 12}

Set B = {2, 3, 4, 6, 7, 9, 11, 12, 13}

The overlapping section represents  $A \cap B = \{6, 7, 9, 12\}$

(The intersection of A and B)

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7) Independent events	One event happening does not affect the probability of the other.
Probability of two independent events	$P(A \text{ and } B) = P(A) \times P(B)$ .
Tree diagram	Shows two or more events and their probabilities.
Dependent events	If one event depends upon the outcome of another event.

Multiply along the branches	Add down the column.
Multiply if you say AND	Add if you say OR.

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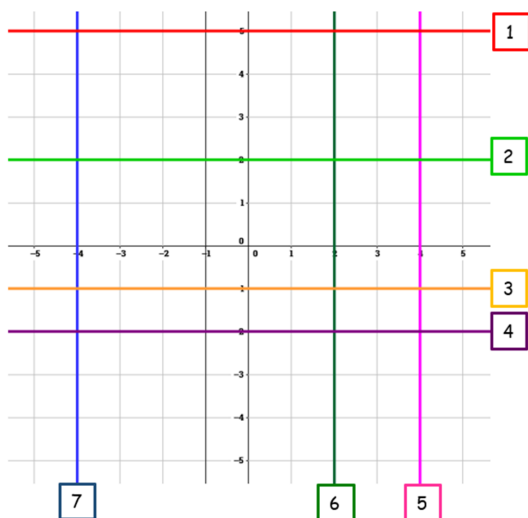
1)	<ul style="list-style-type: none"> <li>An angle measured in degrees</li> <li>Always measure from North</li> <li>Always measure clockwise</li> <li>Write bearings in three figures, eg 215° or 059°.</li> </ul>
Examples	<p>The bearing of B from A is 065°</p> <p>The bearing of A from B is 245°</p>
492+	
Scale	The scale on a map is given as a ratio 1 : n. For example, 1 : 25000 means 1cm on the map is 25000cm or 250m in real life. There are no units included in the scale.

3)	<ul style="list-style-type: none"> <li>Similar shapes <ul style="list-style-type: none"> <li>One shape is an enlargement of the other.</li> <li>Corresponding angles are equal and corresponding sides are all in the same ratio.</li> </ul> </li> </ul>
Scale factors	<p>When the linear scale factor is <math>k</math>:</p> <ul style="list-style-type: none"> <li>Lengths are multiplied by <math>k</math></li> <li>Area is multiplied by <math>k^2</math></li> <li>Volume is multiplied by <math>k^3</math></li> </ul>
	615+

Congruent shapes	Exactly the same size and shape. All side lengths and angles are the same.
Congruent triangles	<p>Two triangles are congruent when one of these conditions of congruence is true.</p> <ul style="list-style-type: none"> <li>SSS (all three sides equal)</li> <li>SAS (two sides and the included angle are equal)</li> <li>ASA (two angles and the included side are equal)</li> <li>RHS (right angle, hypotenuse and one other side are equal)</li> </ul>
Similar triangles	If two triangles have all the same angles (AAA) then they are similar, but not necessarily congruent as the sides may be different lengths.

4) Things to learn to prepare you for GCSE:	<ul style="list-style-type: none"> <li>The quadratic formula solves quadratic equations of the form <math>ax^2 + bx + c = 0</math></li> </ul> $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$									
Trigonometric functions are used to calculate sides and angles.										
	<table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>SOH</td></tr> <tr><td><math>\sin \theta = \frac{O}{H}</math></td></tr> <tr><td><math>\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}</math></td></tr> </table> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>CAH</td></tr> <tr><td><math>\cos \theta = \frac{A}{H}</math></td></tr> <tr><td><math>\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}</math></td></tr> </table> <table border="1" style="display: inline-table;"> <tr><td>TOA</td></tr> <tr><td><math>\tan \theta = \frac{O}{A}</math></td></tr> <tr><td><math>\tan \theta = \frac{\text{opposite}}{\text{adjacent}}</math></td></tr> </table>	SOH	$\sin \theta = \frac{O}{H}$	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$	CAH	$\cos \theta = \frac{A}{H}$	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	TOA	$\tan \theta = \frac{O}{A}$	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
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The Sine Rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ <p>(finding sides)</p>									
The Cosine Rule	$a^2 = b^2 + c^2 - 2bc \cos A$ <p>(finding angles)</p>									

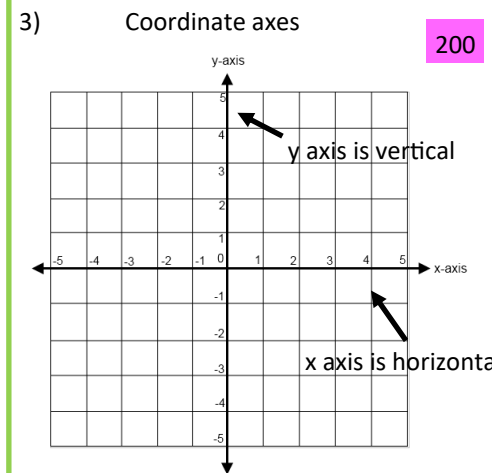
1) Equation of a horizontal line	All points have the same y coordinate, eg $y = 3$ . The line is parallel to the x axis.
Equation of a vertical line	All points have the same x coordinate, eg $x = -5$ . The line is parallel to the y axis.
Drawing graphs	Write the numbers on the axes on the grid lines, not in the middle of the squares. Plot the coordinate points then draw a line right to the edge of the grid with a ruler through the points.
Label a graph	Write the equation of the line next to the line.

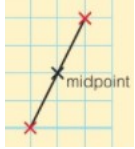


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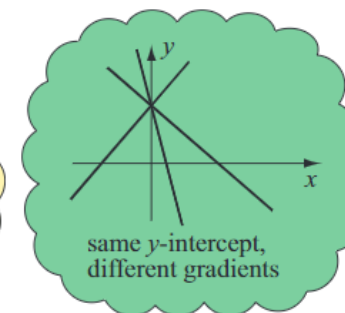
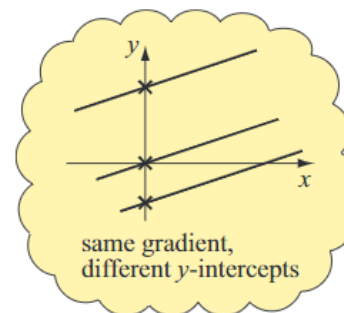
Lines 1- 4 are horizontal. They intersect with the y axis and all begin  $y = \dots$

Lines 5-7 are vertical. They intersect with the x axis and all have equation  $x = \dots$



Line segment	The part of a line that connects two points.
Midpoint	The point exactly in the middle. 
Finding the midpoint	$\left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$

4) Gradient	The steepness of a line. <span style="float: right;">206+</span>
Calculating the gradient	Change in y $\div$ Change in x
Linear equation	Generates a straight line graph.
Equation of a straight line	$y = mx + c$
m	The gradient of the line is given by the coefficient of x.
Coefficient	Number in front of the x.
c	The y intercept.
Intercept	Where the line intersects with the axis.



2) <b>Parallel Lines</b>	Lines that are the same distance apart for all their length.
<b>Perpendicular</b>	Lines that cross or meet at right angles.
<b>Intersecting Lines</b>	Two lines that cross each other are called intersecting lines. The point at which they cross is an intersection.