Mathematics Theta 3 Unit 1

1.1 INDICES

Key Concepts

To multiply powers, add the indices
To multiply powers, add the indices
To work out the power of a power, multiply the indices
A number with a negative power, is the same as the reciprocal of that number to the positive power

Key Concepts

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^{-m} = \frac{1}{a^m}$

Examples

Simplify each of the following:

1) $a^6 \times a^4 = a^{6+4}$ $= a^{10}$
2) $3^6 \times 3^5 = 3^{6+5}$ $= 3^{11}$
3) $a^6 \div a^4 = a^{6-4}$ $= a^2$
4) $9^6 \div 9^3 = 9^{6-3}$ $= 9^3$
5) $(a^6)^4 = a^{6\times4}$ $= a^{24}$
6) $(3a^4)^3 = 3^3a^{4\times3}$ $= 27a^{12}$
7) $a^{-1} = \frac{1}{a^1}$
8) $a^{-2} = \frac{1}{a^2}$

Questions

1) $a^3 \times a^2$ 2) $b^4 \times b$ 3) $d^{-5} \times d^{-1}$
4) $m^6 \div m^2$ 5) $n^4 \div n^4$ 6) $\frac{8^4 \times 8^5}{8^6}$ 7) $\frac{4^9 \times 4}{4^3}$

Evaluate :

1) $(3^2)^5$ 2) 2^{-2} 3) $81^{\frac{1}{2}}$ 4) $27^{\frac{1}{3}}$

Key Words

Powers
Roots
Indices
Reciprocal

ANSWERS: 1) a^5 2) b^5 3) d^{-6} 4) m^4 5) 1 6) 8^3 7) 4^7
1) 3^{10} 2) $\frac{1}{4}$ 3) 9 4) 3

MyMaths
Indices1
Indices 2

MathsWatch
N25

1.2 STANDARD FORM

Key Concepts

- Standard form is used to write very large or small numbers
- A number written in standard form is a number between 1 and 10 and is multiplied by a power of 10 e.g. 4.6×10^3 or 6.7×10^{-5}
- Positive values of b give large numbers greater than 1
- Negative values of b give small numbers less than 1

Must be $\times$ 10
b is an integer

$a \times 10^b$

Must be $1 \leq a < 10$

Examples

Write the following in **standard form**:

1) 3000 = 3×10^3
2) 4580000 = 4.58×10^6
3) 0.0006 = 6×10^{-4}
4) 0.00845 = 8.45×10^{-3}

Questions

A) Write the following in standard form:
1) 74 000 2) 1 042 000 3) 0.009 4) 0.000 001 24
B) Work out:
1) $(5 \times 10^2) \times (2 \times 10^5)$ 2) $(4 \times 10^3) \times (3 \times 10^8)$
3) $(8 \times 10^6) \div (2 \times 10^5)$ 4) $(4.8 \times 10^2) \div (3 \times 10^4)$

Key Words

Standard form
Base 10

ANSWERS: A1) 7.4×10^4 2) 1.042×10^6 3) 9×10^{-3} 4) 1.24×10^{-6}
B1) 1×10^8 2) 1.2×10^{12} 3) 4×10 4) 1.6×10^{-2}

MyMaths
Standard form large
Standard form small

MathsWatch
N45a, N45b,

1.3 ROOTS AND ESTIMATION

Roots

A square root of a number is a value that can be **multiplied by itself 2 times** to give the original number.
The square root of 36 ($\sqrt{49}$) is 7, because when 7 is squared you get 49.

The cube root of a number is a special value that **multiplied by itself 3 times** gives the original number.
The cube root of 27 ($\sqrt[3]{27}$) is 3, because when 3 is cubed you get 27.

Estimation

Estimation is a result of rounding to one significant figure.

a) 1 significant figure
3.27 3

b) 1 significant figure
45 50

c) 1 significant figure
0.075
0.08

Examples

A) Work out the roots of the following numbers
1) $\sqrt{25} = 5$
2) $\sqrt{64} = 8$
3) $\sqrt[3]{27} = 3$
4) $\sqrt[3]{-1} = -1$

Estimate the answer to the following calculation:

$\frac{46.2 - 9.85}{\sqrt{16.3 + 5.42}}$

$\frac{50 - 10}{\sqrt{20 + 5}}$

$\frac{40}{5} = 8$

Questions

A) Work out the roots of the following numbers
1) $\sqrt{36}$ 2) $\sqrt{121}$ 3) $\sqrt[3]{8}$ 4) $\sqrt[3]{-8}$

B) Estimate:
1) $\sqrt{4.09 \times 8.96}$
2) 2) $25.76 - \sqrt{4.09 \times 8.96}$
3) $\sqrt[3]{26.64} + \sqrt{80.7}$ 4) $\frac{\sqrt{6.91 \times 9.23}}{3.95^2 \div 2.02^3}$

Key Words

Negative
Root
Estimate
Significant figure

ANSWERS
A 1) 6 2) 11 3) 2 4) -2
B 1) 6 2) 24 3) 12 4) 4

MyMaths
Estimating calculation

MathsWatch
N25 N43

1.4 SUBSTITUTION

Key Concepts

- When **substituting** a number into an expression, replace the letter with the given value.
- Do not forget to use BIDMAS. This tells you the order in which to work out the question.

Rules for negative numbers:

- Two like signs become positive sign (2 - - 2 = +4) (-2 x -10 = +20)
- Two unlike signs become a negative sign (10 + - 2 = 8) (-2 x 10 = -20)

Brackets
Indices
Division
Multiplication
Addition
Subtraction

ORDER

Follow the order of BIDMAS to calculate

If you can do nothing for a given step, move on to the next

Examples

Find the value of 2x when x = -3
(-2 x 3) = -6

Find the value of $3x$ + 2 when x = 5
(3 x 5) + 2 = 17

Where A = b² + c, find A when b = 2 and c = 3
A = 2² + 3
A = 4 + 3
A = 7

Questions

1) Find the value of 4y when y = 7
2) Find the value of $5x$ – 7 when x = 3
3) Find the value of 10 – 2x when x = -1
4) Where A = d² + e, find A when d = 5 and e = 4
5) Find the value of 3x² when x= 2

Key Words

Substitute
Formula
Expression

ANSWERS:
1) 28 2) 8 3) 12
4) A = 27 5) 12

MyMaths
Substitution 1

MathsWatch
A10

Mathematics Theta 3 Unit 2

2.1 Writing Expressions and Formulas

Key Concept

- Algebra can be used to help us to find out problems in a real life.
- We can always apply a letter to an unknown quantity, to set up an equation/expression.
- Formulae can help us describe situations that work in the same way for different numbers.
- E.g. Cost of hiring a vehicle depends on its type, how long you have it and how far you travel.

Worked example

You can work out a waiter's daily pay using the number of hours worked, h, the hourly rate of pay, r, the total amount of tips, t, and the number of staff who share the tips, s.

a Write an expression for the daily pay in terms of h, r, t and s.

$hr + \frac{t}{s}$

hourly rate × number of hours worked + $\frac{\text{total amount of tips}}{\text{number of staff who share the tip}}$

b Write a formula for the daily pay, P, in terms of h, r, t and s.

$P = hr + \frac{t}{s}$

The formula has 'P =' in front of the expression.

Questions

1) Use algebra to show:
   a. Two less than x
   b. Twice x
   c. Half x
   d. 2 more than double x

2) An electrisation charges £30 per hour and £20 call out fee. Write a formula for the total charge, C, when the plumber is called out for h hours.

ANSWERS: 1a) x -2   1b) 2x
1c) $\frac{x}{2}$ 1d) 2x + 2
2) C = 30 h + 20

MyMaths
Indices 1 and 2

MathsWatch
N25

Key Words
Solve
Term
Inverse operation

2.2 Using Expressions and Formulas

Key Concept

- A formula is a rule that shows a relationship between two or more variables (letters).
- You can use substitution to find each unknown value.

Worked example

Use the formula $v = u + at$ to work out the value of t when $v = 30$, $u = 10$ and $a = 4$

$v = u + at$
$30 = 10 + 4t$
$30 - 10 = 4t$
$20 = 4t$
$\frac{20}{4} = t$
$t = 5$

Substitute the numbers that you know into the formula.
Solve the equation, one step at a time, to find the value of t.

Solve to find the value of x when the perimeter is 42cm.

2x + 3

x

HINT: Write on all of the lengths of the sides.

x

$2x + 3$

We know the perimeter is 42cm

$2x + 3 + 2x + 3 + x + x = 42$
$9x + 6 = 42$
$6x = 36$
$x = 6$

Questions

1) Cost = 30 x hours + £25

The plumber uses this formula to work out his bills. Find the cost of a plumber when he works
a) 3 hours
b) 6 hours

$3x+5$

$x+3$

2) If the perimeter is 40cm. What is the length of the longest side?

ANSWERS:
1a) C = 115
1b) C= 205
2) x = 3

MyMaths
Rules and formulae

MathsWatch
A3, A13

Key Words
Solve
Term
Inverse operation

2.2 Equations

Key Concepts

Solving equations:
Working with inverse operations to find the value of a variable.

Rearranging an equation:
Working with inverse operations to isolate a highlighted variable.

In solving and rearranging we **undo the operations** starting from the last one.

Examples

Solve:

$x - 15 = 8$
+15   +15
$x = 23$

Rearrange to make r the subject of the formulae :

$Q = r - 7$
+7   +7
$Q + 7 = r$

Solve:

$4p - 5 = 3$
+5   +5
$4p = 8$
÷ 2   ÷ 2
$p = 2$

Rearrange to make c the subject of the formulae :

$a = 7c + 1$
-1   -1
$a - 1 = 7c$
÷ 7   ÷ 7

$\frac{a-1}{7} = c$

Questions

1) Solve 2x - 10 = + 4
2) Solve 2 = 5x – 3
3) Rearrange to make m the subject p = 3 + m
4) Rearrange to make x the subject y = 4 + 3x

ANSWERS:
1) $x = 7$ 2) $x = 1$
3) p -3 = m
4) $\frac{y-4}{3} = x$

Key Words
Solve
Rearrange
Term
Inverse

MyMaths
Rearranging Equations

MathsWatch
A13, A12

2.4 EXPANDING BRACKETS AND FACTORISING

Key Concepts

Expanding brackets
Multiply the number outside the brackets with EVERY term inside the brackets

Factoring expressions
Take the highest common factor outside the bracket.

Expand and simplify:

1) $(p + 2)(2p - 1)$
$= p^2 + 4p - p - 2$
$= p^2 + 3p - 2$

3) $(p + 3)(p - 1)$
$= (p^2 + 3p - p - 3)$
$= p^2 + 2p - 3$

Factorise fully:

1) $16at^2 + 12at = 4at(4t + 3)$

2) $x^2 - 2x = x(x - 2)$

Examples

Expand and simplify where appropriate

1) 7 (3 + a)
= 21 + 7a

2) 2(5 + a) + 3(2 + a)
= 10 + 2a + 6 + 3a
= 5a + 16

3) Factorise $9x + 18$
= $9(x + 2)$

4) Factorise $6e^2 - 3e$
= $3e(2e - 1)$

Questions

1) Expand and simplify
(a) 3(2 – 7f)   b) 5(m – 2) + 6
(c) (4 + t) + 2(5 + t)   (d) (p + 3) (p – 2)

2) Factorise
(a) 6m + 12t   (b) 9t – 3p   (c) $4d^2 - 2d$
(d) $4y^2 + 8y$ (e) $3p + 2p^2$

ANSWERS: 1) (a) 6 – 21t   (b) 5m – 4   (c) 22 + 5
(d) p^2 + p -6
2) (a) 6(m +2t)   (b) 3(3t – p)   (c) 2d(2d – 1)
(d) 4y (y + 2)   (e) p (3 + 2p)

MyMaths
Brackets
Factorising linear

MathsWatch
A8, A9

3.1 Planning a survey

Key concepts

Population – the whole group you are interested in
Sample – a smaller group chosen from the population

Advantages of sampling
- Cheaper to survey a sample
- Less time consuming

Ideal sample
- At least 10% of population
- Chosen at random to reduce bias ie every member of the population has an equal chance of being included.

Primary data you collect yourself eg experiment, survey
Secondary data is collected by someone else eg find on internet, in books/newspapers

A good survey question should not be
- vague
- leading
- restrictive

Key point

Discrete data can only take particular values. For example, dress sizes can only be even numbers. For discrete data you can use groups like 1–10, 11–20, …
Continuous data is measured and can take any value. Length, mass and capacity are continuous. For continuous data there are no gaps between the groups. You must use the ⩽ and < symbols.

Literacy hint

A **leading question** encourages people to give a particular answer.

Worked example

Here are three questions used in an online survey.
Explain what is wrong with each question and rewrite it.

a How old are you?
☐ 0–10 ☐ 10–20 ☐ 20–30 ☐ 30–50 ☐ 50+

The groups overlap. For example, if you are 20 years old, which box do you tick?
Change to: ☐ 0–10 ☐ 11–20 ☐ 21–30 ☐ 31–50 ☐ 51+

b Do you agree that exercise is enjoyable? ☐ Yes ☐ No

Saying, 'Do you agree?' encourages the answer 'Yes'.
Change to: Do you enjoy exercise?

c Do you exercise enough?

'Enough' is not precise and means different things to different people.
Change to: How much do you exercise each day?
☐ Less than 1 hour ☐ 1–2 hours ☐ More than 2 hours

3.2 Collecting data

Key point

A **grouped frequency table** has 4 or 5 equal width classes. You can add a tally column for recording the data.

Time taken. T (mins)	Frequency
$0 < t \leq 1$	8
$1 < t \leq 1.5$	22
$1.5 < t \leq 2$	32
$2 < t \leq 3$	19
$3 < t \leq 4$	19
	Total = 100

Each **class** is separated using more than or less than sighs.

How often each number appears in each group is in the **Frequency** column.

Key point

A **two-way table** shows data sorted in two ways, e.g. gender and age.

	English	Maths	Science	Total
Girls	20	13		50
Boys		15		
Total	38		40	

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3.2 Calculating Averages

Worked example

In a survey, people were asked their age. The table shows the results.

Age, a (years)	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11

a Work out an estimate for the range of ages.

From the frequency table, the smallest possible age is 0 years.
The largest possible age is 40 years.
So an estimate of the range is $40 - 0 = 40$ years.

b Calculate an estimate for the mean age.

Age, a (years)	Frequency	Midpoint of class	Midpoint × Frequency
$0 \leqslant a < 10$	12	$\frac{0+10}{2} = 5$	$5 \times 12 = 60$
$10 \leqslant a < 20$	15	$\frac{10+20}{2} = 15$	$15 \times 15 = 225$
$20 \leqslant a < 30$	2	25	$25 \times 2 = 50$
$30 \leqslant a < 40$	11	35	$35 \times 11 = 385$
Total	40		720

Add a column to calculate the midpoint of each class. This represents the ages, because you don't know the exact values in each class.

Add a column to calculate an estimate of the total age for each class.

Calculate the total number of people in the survey and the sum of their ages.

mean = sum of ages ÷ total number of people

$= \frac{720}{40}$

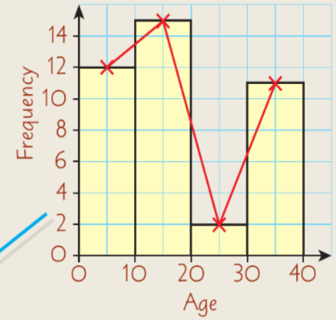
$= 18$

3.4 Frequency Polygon

Worked example

Draw a frequency polygon to represent this data.

Age, a	Frequency
$0 \leqslant a < 10$	12
$10 \leqslant a < 20$	15
$20 \leqslant a < 30$	2
$30 \leqslant a < 40$	11



First draw a frequency diagram. Then join the midpoints of the tops of bars.

Key point

You can draw a **frequency polygon** by joining the midpoints of the tops of the bars in a frequency diagram.

3.5 Writing a report

Key point

A report should include

- the hypothesis or what you are investigating
- the data shown in a graph or chart
- averages and range
- a conclusion
- what else you could investigate.

The Data Handling Cycle

1. Specify the problem and Plan: Ask a question. Then decide what data to collect and how you will collect it.

2. Collecting data: Gather the appropriate data as quickly and efficiently as possible.

3. Processing and presenting the data: Reduce the raw data to summary information, including lists, tables and charts to help to answer the question.

4. Interpret and discuss the results: Use results to answer the initial question and consider further questions.

Mathematics Theta 3 Unit 4

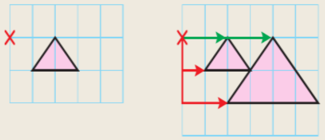
4.1 Enlargement

Literacy hint

We still use 'enlarge' for fractional scale factors, even though they make the shape smaller!

Worked example

Enlarge this triangle using a scale factor 2 and the marked centre of enlargement.



Multiply all the distances from the centre by the scale factor. Count the squares from the centre of enlargement:

- The top vertex of the triangle changes from 2 right to 4 right.
- The bottom left vertex changes from 1 down and 1 right to 2 down and 2 right.

Key point

When you enlarge a shape by a scale factor from a **centre of enlargement**, the distance from the centre to each point on the shape is also multiplied by the scale factor.

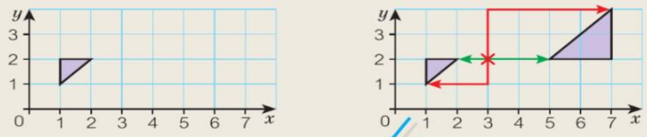
Key point

To describe an enlargement, give the scale factor and the coordinates of the centre of enlargement .

4.2 Negative and scale enlargement

Worked example

Enlarge this triangle using scale factor −2 and centre of enlargement (3, 2).



Count the squares from the centre of enlargement:

- The top right vertex of the small triangle changes to the bottom left vertex of the enlarged triangle, from 1 left to 2 right.
- The bottom vertex of the triangle changes to the top vertex of the enlarged triangle, from 1 down and 2 left to 2 up and 4 right.

Key point

A **negative scale factor** has the same effect as a positive scale factor except that it takes the image to the opposite side of the centre of enlargement.

Key point

You can enlarge a shape using a **fractional scale factor**. Use the same method of multiplying the length of each side by the scale factor.

4.3 Rate of Change

Key point

Compound measures combine measures of two different quantities. For example, speed is a measure of distance travelled and time taken. It can be measured in metres per second (m/s), kilometres per hour (km/h) or miles per hour (mph). You can calculate **average speed** if you know the **distance** and the **time**.

Speed = $\frac{\text{Distance}}{\text{Time}}$ or $S = \frac{D}{T}$

Key point

Pressure is a compound measure. Pressure is the **force** applied over an **area**.

pressure = $\frac{\text{force}}{\text{area}}$ or $P = \frac{F}{A}$

Pressure is usually measured in newtons (N) per square metre. To calculate it, you need pressure in N and area in m^2.

Key point

Density is a compound measure. Density is the **mass** of substance contained in a certain **volume**. To calculate it, you need mass in g and volume in cm^3.

density = $\frac{\text{mass}}{\text{volume}}$ or $D = \frac{M}{V}$

Density is usually measured in grams per cubic centimetre (g/cm^3).

Example

A silver pendent has a mass of 31.5 g and a volume of 3cm3. Work out the density of the silver

$D = \frac{M}{V}$ $D = \frac{31.5}{3}$ D = 10 . 5 g/cm3

The speed of a car was 10m/s and travelled for 10s at this speed, what was the distance travelled?

(We need to change the subject of the formula to find out the distance)

D = S x T D = 10 x 10 D = 100 m

Questions

1) **A car travels for 120 miles and takes 3 hours to get there. Work out the speed of the car.**
2) **Work out the pressure of the water when the area is 3m3 and the force is 12 N.**
3) **Find the mass of a rock when the density is 12g/m3 and the volume is 5m3.**

Key Words

Compound Measure
Density
Pressure

Answers

1) 40 miles per hour
2) 4 N
3) 60 grams

MyMaths

Converting compound measure

MathsWatch

R11a, R11b

4.4 Percentage Change

Percentage change:

You can find the change in percentage by using the formula:

Change in percentage = $\frac{\textit{actual change}}{\textit{oringal amount}}$ ***X* 100**

A dress is reduced in price from £50 to £40. What is the percentage change?

Change in percentage = $\frac{10}{50}$ ***X* 100 = 20% decrease**

A house price increases in a year from 120,000 to 150,000. What is the **percentage change**?

Change in percentage = $\frac{30000}{120000}$ ***X* 100**

= 25% increase

Reverse percentages:

This is when we are trying to find out the original amount.

There are two ways you can do this:

A TV was bought in a sale for £120, the sign says there is 20% off. What was it worth **originally**?

100% - 20% = 80% = 0.8

Original *Value* → x 0.8 → £120

£ 150 ← ÷0.8 ← £120

A pair of trainers cost £35 in a sale. If there was 20% off, what was the **original price** of the trainers?

$Value \div (1 - 0.20)$
= 35 ÷ 0.8
= £43.75

MyMaths

Percentage Change 1

MathsWatch

N39a

1a) Shoes were £25 but are now £15. What is the percentage change?
2) A camera costs £180 in a 10% **sale**. What was the **pre-sale** price
3) The cost of a holiday, was reduced by 20% and is now £540. What is the **pre-sale** price?

Key Words

Percent
Increase/decrease
Reverse
Multiplier
Inverse

ANSWERS
1) 60% decrease
2) £200 3) £675

5.1Using Scales

Key Concept
- A drawing that shows a real object with accurate sizes reduced or enlarged by a certain amount (called the scale).
- The scale is shown as the length in the drawing, then a colon (":"), then the matching length on the real thing.
- Example: in a drawing anything with a scale of 1:10, the size of "1" would have a size of "10" in the real world, so a measurement of 10mm on the drawing would be 100mm in real life.

Worked example
A map has a scale of 1 : 25000.
What distance in metres does 3 cm on the map represent?

Map		Real life
1 cm	is	25 000 cm
3 cm	is	75 000 cm

×3 ×3

1 cm represents 25 000 cm,
3 cm represents 3 × 25 000 = 75 000 cm. Work out the real life distance in cm.
75 000 cm ÷ 100 = 750 m Convert to metres

Questions
1) A map has the scale of 1:25000
a) What distance in centimeters does 5cm on the map represent?
b) What distance in kilometers does 8cm on the map represent?
c) In real life the distance is 25km, how far would this be on the map?

MyMaths
Map Scales
MathsWatch
R6

Key Words
Ratio
Scale
Drawings
Accurate

Answer
1c)
1b) 2km
1a) 1250 cm

5.2 Construction

Key Concept
- Constructions are accurate diagrams drawn using a pair of compasses and a ruler.
- When drawing constructions, the construction lines must not be rubbed out.
- You can solve problems using constructions
- Problems can involve intersecting loci. It may be necessary to use several constructions to locate a region.

Questions
1) Can you draw a
a) Perpendicular bisector?
b) Angle bisector?
c) Perpendicular bisector through a point?

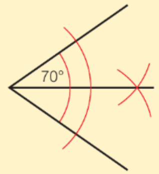
Use worked examples to check your work.

Key Words
Perpendicular bisector
Angle bisector
Compass
Construct

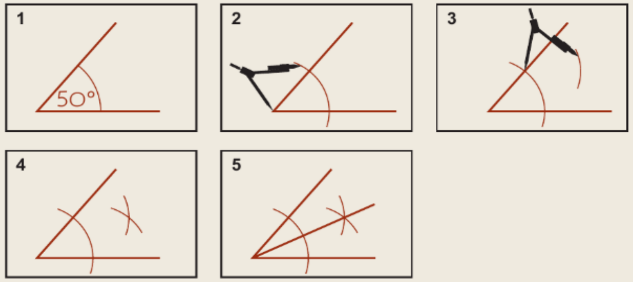
MyMaths
Construction triangles/loci
MathsWatch
G26

Key point
Construct means draw accurately using ruler and compasses.
A **perpendicular bisector** cuts a line in half at right angles.

Key point
An **angle bisector** cuts the angle in half.



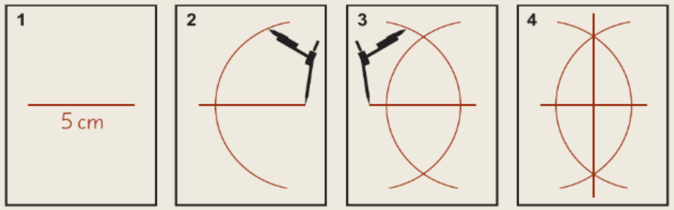
Worked example
Draw an angle of 50°. Construct the angle bisector.



1 Draw the angle using a protractor.
2 Open your compasses and place the point at the vertex of the angle. Draw an arc that cuts both arms of the angle.
3 Keep the compasses the same. Move them to a point where the arc crosses one of the arms. Make an arc in the middle of the angle.
4 Do the same from the point where the arc crosses the other arm.
5 Join the point where the arcs cross to the vertex of the angle.
The line joins the point where the two small arcs intersect to the point of the angle; it divides the angle exactly in half.

Literacy hint
Bisect means cut in half.

Worked example
Draw a line that is 5 cm long.
Construct its perpendicular bisector.

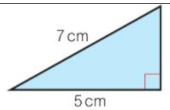


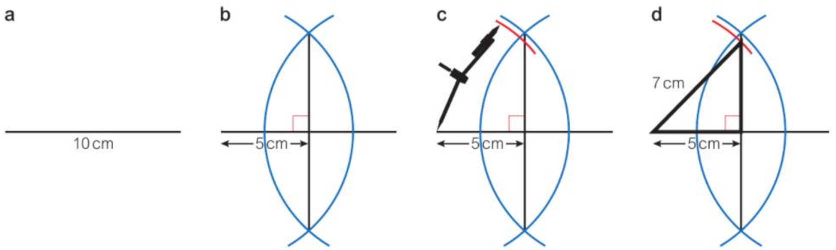
1 Use a ruler to draw the line.
2 Open your compasses greater than half the length of the line. Place the point on one end of the line and draw an arc above and below.
3 Keeping the compasses the same, move them to the other end of the line and draw another arc.
4 Join the points where the arcs intersect. The vertical line divides the horizontal line exactly in half.
Do not rub out the arcs.

Mathematics Theta 3 Unit 5

5.3 Constructing Triangles

Follow these instructions to construct this right-angled triangle.

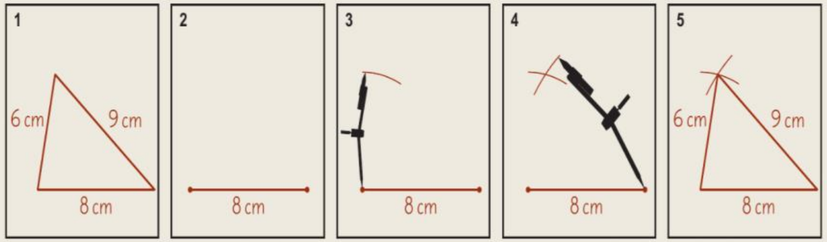




- **a** Draw a straight line twice the length of the base.
- **b** Construct the perpendicular bisector.
- **c** Open your compasses to 7 cm (for the sloping side). Put the point of your compasses at the end of your base line. Draw an arc to cut the vertical line.
- **d** Join the points.

Worked example

Construct a triangle with sides of 8 cm, 6 cm and 9 cm.



1. Sketch the triangle first.
2. Draw an 8 cm line.
3. Open your compasses to 6 cm. Place the point at one end of the 8 cm line. Draw an arc.
4. Open the compasses to 9 cm. Draw an arc from the other end of the 8 cm line.
5. Join the intersection of the arcs to each end of the 8 cm line.

5.4 Loci

Key Concept

- The word locus describes the position of points which obey a certain rule and this usually results in a curve or surface.

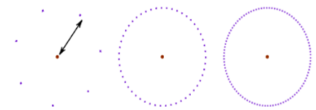
Three important loci are the following:

- **The circle:** the locus of points which are equidistant from a fixed point, the centre.
- **The perpendicular bisector:** the locus of points which are equidistant from two fixed points, A and B.
- **The angle bisector:** the locus of points which are equidistant from two fixed lines.

Examples

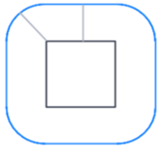
A circle is the locus of points that are a certain distance from a central point.

Just a few points start to look like a circle, but when we collect ALL the points we will actually have a circle.



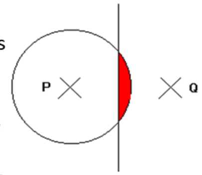
If a point P is 'equidistant' from two points A and B, then the distance between P and A is the same as the distance between P and B, as illustrated here: the points on the line are equidistant from A and B.

A set distance from a straight line will give semi-circles around the ends and parallel lines connecting them.

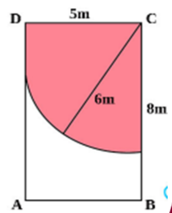


A set distance around a square will give quarter-circles around the corners and parallel lines to the square's sides.

The diagram shows two points P and Q. On the diagram, the shaded region contains all the points which satisfy both the following: the distance from P is less than 3cm; the distance from P is greater than the distance from Q.



The walls of a rectangular shed, ABCD, measure 8m by 5m. A goat is tied to the corner of C by a rope 6m long. The shaded area shows the part the goat can reach.



Mathematics Theta 3 Unit 6

6.1 Solving Equations

Key Concepts

Solving equations:

- Working with inverse (opposite) operations (+ - x ÷) to find the value of a variable (a, b x, y).

Rearranging an equation:

- For each step in solving an equation, we must do the **inverse** operation.
- In solving and rearranging we **undo the operations** starting from the last one.

Inverse operations is where you find the opposite sign to the one you have.
3a means 3 x a
So we divide by 3 to get a single a
3a ÷ 3 = a

Solve:

$5(x-3) = 20$
Expand
$5x - 15 = 20$
+15 +15
$5x = 35$
÷5 ÷5
$x = 7$

Solve:

$12 = 3x - 18$
+18 +18
$30 = 3x$
÷3 ÷3
$x = 10$

Rearrange to make r the subject of the formulae :

$Q = \frac{2r-7}{3}$
×3 × 3
$3Q = 2r - 7$
+7 +7
$3Q + 7 = 2r$
÷ 2 ÷ 2
$\frac{3Q+7}{2} = r$

Questions

1) Solve $7(x + 2) = 35$
2) Solve $4x - 12 = 28$
3) Solve $4x - 12 = 2x + 20$
4) Rearrange to make x the subject:
$y = \frac{3x+4}{2}$

ANSWERS: 1) $x = 3$ 2) $x = 10$ 3) $x = 16$ 4) $x = \frac{2y-4}{3}$

MyMaths
Algebra-Equations Linear
MathsWatch
A12, A19

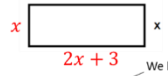
Key Words
Solve
Term
Inverse
Rearrange

6.2 Equations in context

Examples

Solve to find the value of x when the perimeter is 42cm.

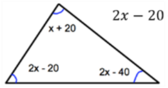
HINT: Write on all of the lengths of the sides.



We know the perimeter is 42cm

$2x + 3 + 2x + 3 + x + x = 42$
$9x + 6 = 42$
$6x = 36$
$x = 6$

Angles in a triangle sum to 180°



$2x - 20 + x + 20 + 2x - 40 = 180$
$5x - 40 = 180$
$5x = 220$
$x = 45$

Jane is 4 years older than Tom.
David is twice as old as Jane.
The sum of their ages is 60.
Using algebra, find the age of each person.

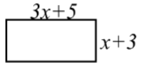
Tom = x → 12
Jane = $x + 4$ → $12 + 4 = 16$
David = $2x + 8$ → $(2 \times 12) + 8 = 32$

$x + x + 4 + 2x + 8 = 60$
$4x + 12 = 60$
$4x = 48$
$x = 12$

Key Concepts

- Algebra can help us find unknowns in a real-life problem.
- We can always apply a letter to an unknown quantity, to then set up an equation.
- It will often be used in area and perimeter problems and angle problems in geometry.

Questions



1) If the perimeter is 40cm, what is the length of the longest side?

2) Jane is 12 years older than Jack.
Sarah is 3 years younger than Jack.
The sum of their ages is 36.
Using algebra, find the age of each person.

ANSWERS: 1) $x = 3$ therefore the longest length is 14cm 2) Jack = 9, Jane = 21, Sarah = 6

MyMaths
Algebra-Equations Linear
MathsWatch
A17

Key Words
Solve
Term
Inverse
operation

6.3 Trial and Improvement

Key Concepts

- Most equations do not have whole number (integer) solutions and we have to sometimes find an approximate (roughly correct) answer. This method is called **trial and improvement**.
- You will be asked to give an answer to a given number of decimal places or significant figures.
- **Example below**: we are looking for a number, that when applied to the equation will give us 6. Start by guessing what it could be and then refine your answer based on your result.

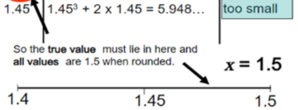
Trial and Improvement

Solve $x^3 + 2x = 6$ (to 1 d.p)

x	$x^3 + 2x$	6
1	$1^3 + 2 \times 1 = 3$	too small
2	$2^3 + 2 \times 2 = 12$	too big
1.3	$1.3^3 + 2 \times 1.3 = 4.797$	too small
1.4	$1.4^3 + 2 \times 1.4 = 5.544$	too small
1.5	$1.5^3 + 2 \times 1.5 = 6.375$	too big
1.45	$1.45^3 + 2 \times 1.45 = 5.948...$	too small

So the **true value** must lie in here and **all values** are 1.5 when rounded.

$x = 1.5$



Midpoint value:
too **big** → round **down**
too **small** → round **up**

The Method

1. Make a table similar to this one. 6 is your **target number**.
2. Make an intelligent guess to find two positive consecutive integers that output values that straddle the target number.
3. Repeat above with consecutive 1 d.p. numbers between 1 and 2.(Trying 1.3)
4. One of these is the correct 1 d.p solution but which one?
5. Compute the midpoint output.

Questions

Use trial and improvement to find an answer to these equations to 1 decimal place:

1) Solve $4x + 6 = 20$
2) Solve $x^2 + 3 = 45$
3) Solve $x^3 + 2x = 10$

ANSWERS:
1) $x = 3.5$ (1d.p)
2) $x = 6.5$ (1d.p)
3) $x = 1.8$ (1d.p)

MyMaths
Trial and Improvement
MathsWatch
A16, A25

Key Words
Solve
Term
Inverse
Operation

6.4 Using and Solving Inequalities

Key Concepts

Inequalities show the **range** of numbers that satisfy a rule.

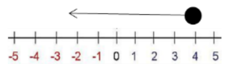
$x < 2$ means x is less than 2
$x \leq 2$ means x is less than or equal to 2
$x > 2$ means x is greater than 2
$x \geq 2$ means x is greater than or equal to 2

On a **number line,** we use circles to highlight the key values:

● is used for less/greater than

○ is used for less/greater than

$x \leq 4$

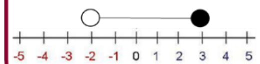


Examples

a) State the values of n that satisfy:
$-2 < n \leq 3$
Cannot be equal to 2 Can be equal to 3

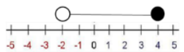
-1, 0, 1, 2, 3

b) Show this inequality on a number line:



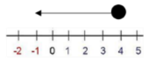
Solve this inequality and represent your answer on a **number line:**

$-2 < x \leq 4$



Solve this inequality and represent your answer on a **number line (treat the inequality like an equals sign):**

$3x + 1 \leq 13$
-1 -1
$3x \leq 12$
÷3 ÷3
$x \leq 4$



Questions

1) State the values of n that satisfy $-3 \leq n < 2$
2) Solve $4x - 2 \leq 6$ and show your answer on a number line.
2) Solve $2x + 3 \leq 9$ and show your answer on a number line.

ANSWERS:
1) -3, -2, -1, 0, 1
2) $x \leq 2$
3) $x \leq 3$

MyMaths
Inequalities
MathsWatch
A20

Key Words
Inequality
Greater than
Less than
Number line

6.4 Using and Solving Inequalities

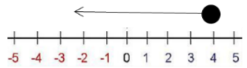
Key Concepts
Inequalities show the **range** of numbers that satisfy a rule.
$x < 2$ means x is less than 2
$x \le 2$ means x is less than or equal to 2
$x > 2$ means x is greater than 2
$x \ge 2$ means x is greater than or equal to 2

On a **number line,** we use circles to highlight the key values:

● is used for less/greater than

○ is used for less/greater than

$x \le 4$

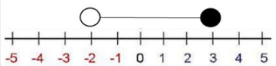


Examples

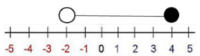
a) State the values of n that satisfy:
$-2 < n \le 3$
Cannot be equal to 2   Can be equal to 3

-1, 0, 1, 2, 3

b) Show this inequality on a number line:

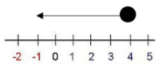


Solve this inequality and represent your answer on a **number line:**
$-2 < x \le 4$



Solve this inequality and represent your answer on a **number line (treat the inequality like an equals sign):**

$3x + 1 \le 13$
$-1 \quad -1$
$3x \le 12$
$\div 3 \quad \div 3$
$x \le 4$



Questions

1) State the values of n that satisfy $-3 \le n < 2$
2) Solve $4x - 2 \le 6$ and show your answer on a number line.
2) Solve $2x + 3 \le 9$ and show your answer on a number line.

ANSWERS:
1) -3, -2, -1, 0, 1
2) $x \le 2$
3) $x \le 3$

MyMaths
Inequalities
MathsWatch
A20

Key Words
Inequality
Greater than
Less than
Number line

6.5 Proportion

Examples

Ingredients for 10 Flapjacks
80 g rolled oats
60 g butter
30 ml golden syrup
36 g light brown sugar

The recipe shows the ingredients needed to make 10 Flapjacks.
How much of each will be needed to make 25 flapjacks?

Method 1: Unitary
$80 \div 10 = 8$
$8 \times 25 =$ **200g**
$30 \div 10 = 3$
$3 \times 25 =$ **75g**
$60 \div 10 = 6$
$6 \times 25 =$ **150g**
$36 \div 10 = 3.6$
$3.6 \times 25 =$ **90g**

Method 2: 5 flapjacks
$80 \div 2 = 40$
$40 \times 5 =$ **200g**
$30 \div 2 = 15$
$15 \times 5 =$ **75g**
$60 \div 2 = 30$
$30 \times 5 =$ **150g**
$36 \div 2 = 18$
$18 \times 5 =$ **90g**

If 20 apples weigh 600g. How much would 28 apples weigh?

$600 \div 20 = 30\text{g} \longrightarrow$ weight of 1 apple

$30 \times 28 =$ **840g**

Box A has 8 fish fingers costing £1.40.
Box B has 20 fish fingers costing £ 3.40.
Which box is the better value?

$A = \frac{£1.40}{8} = £0.175$ $B = \frac{£3.40}{20} = £0.17$

Therefore Box B is better value as each fish finger costs less.

To calculate the **value** for a single item we can use the **unitary method.**

When working with best value in monetary terms we use:
$$Price\ per\ unit = \frac{price}{quantity}$$
In recipe terms we use:
$$Weight\ per\ unit = \frac{weight}{quantity}$$

Questions

Ingredients
to make **16** gingerbread men

180 g flour
40 g ginger
110 g butter
30 g sugar

1) How much will we need to make 24 gingerbread men?

2) Packet A has 10 toilet rolls costing £3.50.
Packet B has 12 toilet rolls costing £3.60.
Which is better value for money?

3) If 15 oranges weigh 300g. What will 25 oranges weigh?

MyMaths
Direct Proportion
MathsWatch
R8

ANSWERS 1) 270g flour, 60g ginger, 165g butter, 45g sugar
2) Packet B 30p per roll
3) 500g

Key Words
Unitary   Best Value
Proportion   Quantity

Mathematics Theta 3 Unit 6

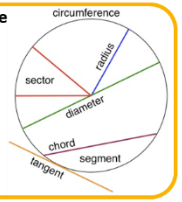
Mathematics Theta 3 Unit 7

7.1 Circumference of a Circle

Key Concepts

- Normally the length around the outside of a shape is called the perimeter, but with circles it is called **circumference.**
- There is a relationship/ratio between the diameter and the circumference, this is called **Pi (π).**
- The circumference of a circle is calculated by πd or $2\pi r$.

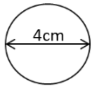
Parts of a circle



Examples

Calculate:

a) **Circumference**

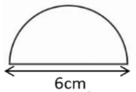


C = $\pi \times 4$
= 4π
or = 12.57cm

b) **Diameter** when the circumference is 20cm

C = $\pi \times d$
20 = $\pi \times d$
$\frac{20}{\pi} = d$
Or $6.37cm$

c) **Perimeter**



$P = \frac{\pi \times d}{2} + d$

$P = \frac{\pi \times 6}{2} + 6$

$P = 3\pi + 6$

Or = 15.42cm

Questions

Calculate:

1) The circumference of a circle with a diameter of 12cm;
2) The diameter of a circle with a circumference of 30cm;
3) The perimeter of a semicircle with diameter 15cm.

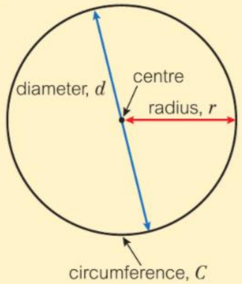
ANSWERS:
1) 12π or 37.7cm
2) $\frac{30}{\pi}$ or 9.54cm
3) 38.56cm

MyMaths
Circumference of a circle
MathsWatch
G22

Key Words
Circle
Perimeter
Circumference
Radius
Diameter
Pi

Key point

The **circumference** (C) is the perimeter of a circle.
The **centre** of a circle is marked using a dot.
The **radius** (r) is the distance from the centre to the circumference.
The plural of radius is **radii**.
The **diameter** (d) is a line from one edge to another through the centre.



Key point

The Greek letter π (pronounced pi) is a special number 3.141 592 653 5…
To find the circumference C of a circle with diameter d, use the formula $C = \pi d$.
If you know the radius r you can use the equivalent formula $C = 2\pi r$.
Use the π key on your calculator.

Key point

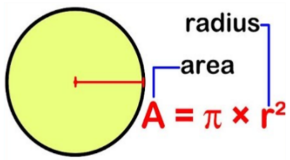
The formula for the area A of a circle with radius r is $A = \pi r^2$.

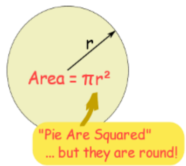
7.2 Area of a Circle

Key Concepts

The **area** of a circle is the space inside the circle.

The area of a circle is calculated by πr^2

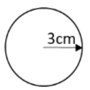




Examples

Calculate:

a) **Area**

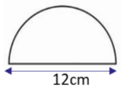


A = $\pi \times 3^2$
= 9π
or = 28.3cm^2

b) **Radius** when the area is 20cm^2

A = $\pi \times r^2$
20 = $\pi \times r^2$
$\frac{20}{\pi} = r^2$
$\sqrt{\frac{20}{\pi}} = r$
Or $2.52cm$

c) **Area**



$P = \frac{\pi \times r^2}{2}$

$P = \frac{\pi \times 6^2}{2}$

$P = 18\pi$

Or = 56.55cm^2

Questions

Calculate:

1) The area of a circle with a radius of 9cm;
2) The radius of a circle with an area of 45cm^2;
3) The area of a semicircle with diameter of 16cm.

ANSWERS:
1) 81π or 254.47cm^2
2) $\sqrt{\frac{45}{\pi}}$ or 3.78cm
3) 32π or 100.53cm

MyMaths
Area of a Circle
MathsWatch
G22

Key Words
Circle
Area
Radius
Diameter
Pi

7.3 Pythagoras

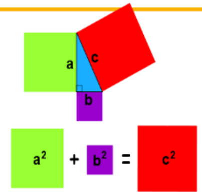
Key Concepts

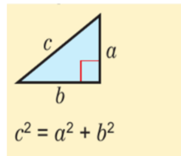
Pythagoras' theorem shows the relationship between the lengths of the three sides of a right-angled triangle.

Pythagoras' Theorem is used to find a missing length when two sides are known.

$a^2 + b^2 = c^2$

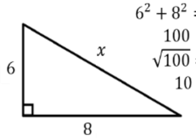
c is always the hypotenuse (longest side)



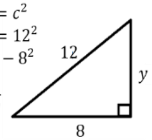


Examples

Pythagoras' Theorem



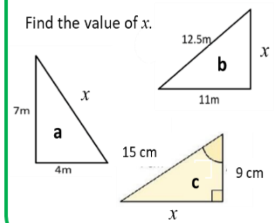
$a^2 + b^2 = c^2$
$6^2 + 8^2 = x^2$
$100 = x^2$
$\sqrt{100} = x$
$10 = x$



$a^2 + b^2 = c^2$
$y^2 + 8^2 = 12^2$
$y^2 = 12^2 - 8^2$
$y^2 = 80$
$y = \sqrt{80}$
$y = 8.9$

Questions

Find the value of x.



ANSWERS:
a) 8.06m
b) 5.64m
c) 12 cm

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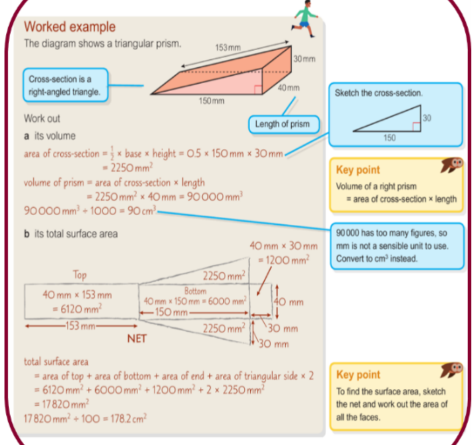
Pythagoras Theorem
MathsWatch
G30

Key Words
Right angled triangle
Hypotenuse

Mathematics Theta 3 Unit 7

7.4 Prisms


Worked example
The diagram shows a triangular prism.
Cross-section is a right-angled triangle.
153 mm
30 mm
40 mm
150 mm
Length of prism
Work out
a its volume
area of cross-section = $\frac{1}{2}$ × base × height = 0.5 × 150 mm × 30 mm
= 2250 mm^2
volume of prism = area of cross-section × length
= 2250 mm^2 × 40 mm = 90 000 mm^3
90 000 mm^3 ÷ 1000 = 90 cm^3
Sketch the cross-section.
30
150
Key point
Volume of a right prism
= area of cross-section × length
90 000 has too many figures, so mm is not a sensible unit to use. Convert to cm^3 instead.
b its total surface area
40 mm × 30 mm
= 1200 mm^2
Top
40 mm × 153 mm
= 6120 mm^2
2250 mm^2
Bottom
40 mm × 150 mm = 6000 mm^2
← 150 mm →
40 mm
← 153 mm →
NET
2250 mm^2
30 mm
30 mm
total surface area
= area of top + area of bottom + area of end + area of triangular side × 2
= 6120 mm^2 + 6000 mm^2 + 1200 mm^2 + 2 × 2250 mm^2
= 17 820 mm^2
17 820 mm^2 ÷ 100 = 178.2 cm^2
Key point
To find the surface area, sketch the net and work out the area of all the faces.

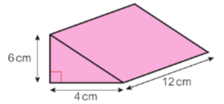
A prism is a solid object with **identical ends**, **flat faces** and the **same cross section** all along its length.

The volume of a prism is the area of one end times the length of the prism.
Volume = Area of cross-section × Length

The surface area of a prism is the area of each of its faces added together.
Surface = 2 × Base Area + Base Perimeter × Length
Area

Questions

The diagram shows a triangular prism.
a Work out the volume.
b Work out the surface area.



Answers
a 144 cm^3
b 231 cm^2

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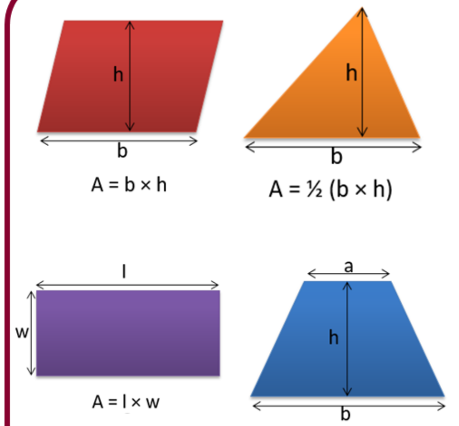


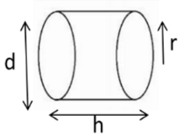
$$A = \frac{(a+b) \times h}{2}$$

Useful to remember these for the faces of prisms.

7.4 Cylinders

Key Concepts

A **cylinder** is a **prism** with the cross section of a circle.

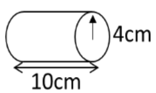


The **volume** of a cylinder is calculated by $\pi r^2 h$ and is the space inside the 3D shape.

The **surface area** of a cylinder is calculated by $2\pi r^2 + \pi dh$ and is the total of the areas of all the faces on the shape.

Examples

From the diagram calculate:



a) **Volume**

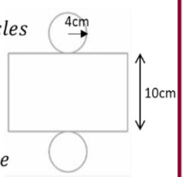
$V = \pi \times r^2 \times h$

$V = \pi \times 4^2 \times 10$

$V = 160\pi$

$\text{Or} = 502.65cm^3$

b) **Surface Area –** You can use the net of the shape to help you



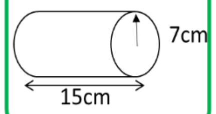
$\textit{Area of two circles}$
$= 2 \times \pi \times r^2$
$= 2 \times \pi \times 4^2$
$= 32\pi$

$\textit{Area of rectangle}$
$= \pi \times d \times h$
$= \pi \times 8 \times 10$
$= 80\pi$

$\textit{Surface Area} = 32\pi + 80\pi$
$= 112\pi \text{ or } = 351.86cm^3$

Questions

Calculate the volume and surface area of this cylinder



ANSWERS: Volume =
735π or $2309.07cm^3$
Surface area = 308π
or $967.61cm^3$

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Key Words
Cylinder
Surface Area
Volume
Prism

7.5 Error Bounds

Key Concepts

The boundaries of a number come from **rounding**.
E.g. State the boundaries of 360 when it has been rounded to the nearest 10: $355 \leq x < 365$

E.g. State the boundaries of 4.5 when it has been rounded to 2 decimal places: 4.45≤x<4.55

These boundaries can also be called the **error bounds** of a number.

When completing calculations involving boundaries, we are aiming to find the greatest or smallest answer.

	+	-	×	÷
Upper bound answer	$UB_1 + UB_2$	$UB_1 - LB_2$	$UB_1 \times UB_2$	$UB_1 \div LB_2$
Lower bound answer	$LB_1 + LB_2$	$LB_1 - UB_2$	$LB_1 \times LB_2$	$LB_1 \div UB_2$

Examples

A restaurant provides a cuboid stick of butter to each table. The dimensions are 30mm by 30mm by 80mm, correct to the nearest 5mm. Calculate the upper and lower bounds of the volume of the butter.

$\textit{Volume} = l \times w \times h$

$\textit{Upper bound} = 32.5 \times 82.5 \times 32.5 = 87140.63mm^3$

$\textit{Lower bound} = 27.5 \times 77.5 \times 27.5 = 58609.38mm^3$

$D = \frac{x}{y}$

x = 99.7 correct to 1 decimal place.
y = 67 correct to 2 significant figures.
Work out an upper and lower bounds for D.

$\textit{Upper bound } D = \frac{99.75}{66.5} = 1.5$ $\textit{Lower bound } D = \frac{99.65}{67.5} = 1.48$

Questions

Jada has 100 litres of oil, correct to the nearest litre.

The oil is poured into tins of 1.5 litres, correct to one decimal place.
Calculate the upper and lower bounds for the number of tins that can be filled.

ANSWERS:
1) LB = 69.3 ≈ 69
UB = 64.2 ≈

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Upper and lower bounds
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G29

Key Words
Bound
Upper
Lower
Rounding